

温度场中功能梯度材料圆板的非线性弯曲

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摘 要:采用弹性理论建立了功能梯度材料板的静力平衡方程, 利用静力平衡方程确定了功能梯度材料板的中性面位置, 在此基础上推导出了功能梯度材料板在温度场中的非线性弯曲微分方程组, 求得了圆板中心挠度与均布载荷的关系式, 讨论分析了梯度指数、温度等因素对功能梯度材料板非线性弯曲的影响. 把该方法计算结果与有限元计算结果进行了比较, 验证了该方法的计算结果是可靠的. 算例分析表明, 中性面位置对温度场中功能梯度材料圆板的非线性弯曲有一定影响, 在荷载较大时, 宜考虑中性面位置对弯曲的影响.

关键词:温度场; 功能梯度材料; 圆板; 非线性; 弯曲

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功能梯度材料(FGM)一般由两种性质完全不同的材料(比如金属和陶瓷)复合而成, 各组分的体积含量连续变化, 由于这类材料的力学和热学参数没有突变, 因而可大大缓解应力集中. 功能梯度的概念是一种全新的材料设计理念, 其应用前景十分广阔. 功能梯度材料是近年来的研究热点^[1-9], 对于陶瓷和金属混合而成的功能梯度材料, 由于陶瓷具有低传热系数而用于抵抗高温, 金属则由于其良好的延展性而防止了短时间内温度剧变产生的应力而导致断裂破坏, 因此被广泛地应用在航空航天等实际工程中. 国内外对功能梯度材料梁、板、壳的研究是很多的, 但在功能梯度材料板的研究中一般假设板相对于中性面具有几何和弹性对称^[9-14], 然后建立功能梯度材料板的弯曲微分方程, 然而一般功能梯度材料板中性面与板中面是不重合的, 这种研究方法显然是具有局限性的. 基于上述原因, 本文首先确定了功能梯度材料板的中性面位置, 建立了功能梯度材料板的非线性弯曲微分方程组, 通过算例, 分析了在不同温度的均匀温度场中, 材料性质的不同梯度变化对平板结构响应的影响.

1 圆板非线性弯曲微分方程

均匀温度场中功能梯度材料板, 板的上面及下面的材料分别为: 陶瓷、金属, 中间则为两种材料组成的混合物, 考虑到金属、陶瓷材料的泊松比相近, 设均为 μ . 设金属材料的弹性模量、热膨胀系数分别为 E_m 、 α_m , 陶瓷材料的弹性模量、热膨胀系数分别为 E_c 、 α_c , 则板内任一点的弹性模量、热膨胀系数分别为

$$E(z) = E_1 V_m + E_c, \alpha(z) = \alpha_1 V_m + \alpha_c \quad (1)$$

式中: $E_1 = E_m - E_c$, $\alpha_1 = \alpha_m - \alpha_c$, V_m 为金属材料组分的体积比例系数.

对于图 1 所示均匀温度场中功能梯度材料板, 令坐标原点在板中性面, 设

$$V_m = \left(\frac{-z_0 + z}{h} + \frac{1}{2} \right)^k \quad (2)$$

式中: k 为梯度指数, z_0 为板中面在 z 轴上的坐标.

根据弹性理论, 功能梯度材料板在均匀温度场中弯曲应力的物理方程为

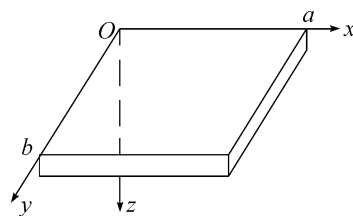


图 1 板的直角坐标系
Fig. 1 Rectangular coordinate system of plate

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$$\begin{cases} \sigma_x = -\frac{E(z)z}{1-\mu^2} \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) - \frac{E(z)\alpha(z)}{1-\mu} \Delta T \\ \sigma_y = -\frac{E(z)z}{1-\mu^2} \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right) - \frac{E(z)\alpha(z)}{1-\mu} \Delta T \\ \tau_{xy} = -\frac{E(z)z}{1+\mu} \frac{\partial^2 w}{\partial x \partial y} \end{cases} \quad (3)$$

式中, ΔT 为温度增量.

当 $\Delta T = 0$ 时, 功能梯度材料板纯弯曲的横截面内力应满足以下关系

$$\int_A \sigma_x dA = 0, \quad \int_A \sigma_y dA = 0 \quad (4)$$

把式(3)代入式(4)中可以得到

$$\int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}} E(z)z dz = 0 \quad (5)$$

把式(1)、式(2)代入式(5)中可得

$$z_0 = \frac{(E_c - E_m)kh}{2(k+2)(E_m + kE_c)} \quad (6)$$

利用式(3)可以得到功能梯度材料板弯矩、扭矩表达式为

$$M_x = -\left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) \int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}} \frac{E(z)z^2}{1-\mu^2} dz - \int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}} \frac{E(z)\alpha(z)z}{1-\mu} \Delta T dz = -D \left(\frac{\partial^2 w}{\partial x^2} + \mu \frac{\partial^2 w}{\partial y^2} \right) - M_T \quad (7a)$$

$$M_y = -\left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right) \int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}} \frac{E(z)z^2}{1-\mu^2} dz - \int_{z_0-\frac{h}{2}}^{z_0+\frac{h}{2}} \frac{E(z)\alpha(z)z}{1-\mu} \Delta T dz = -D \left(\frac{\partial^2 w}{\partial y^2} + \mu \frac{\partial^2 w}{\partial x^2} \right) - M_T \quad (7b)$$

$$M_{xy} = -(1-\mu)D \frac{\partial^2 w}{\partial x \partial y} \quad (7c)$$

式中:

$$\begin{aligned} D &= \frac{E_1}{1-\mu^2} \left[\frac{h}{k+1} \left(z_0 + \frac{h}{2} \right)^2 - \frac{2h^2}{(k+1)(k+2)} \left(z_0 + \frac{h}{2} \right) + \frac{2h^3}{(k+1)(k+2)(k+3)} \right] + \\ &\quad \frac{E_c}{3(1-\mu^2)} \left[\left(z_0 + \frac{h}{2} \right)^3 - \left(z_0 - \frac{h}{2} \right)^3 \right] \\ M_T &= \frac{E_c \alpha_c \Delta T}{2(1-\mu)} \left[\left(z_0 + \frac{h}{2} \right)^2 - \left(z_0 - \frac{h}{2} \right)^2 \right] + \frac{E_1 \alpha_1 h \Delta T}{(2k+1)(1-\mu)} \left(z_0 + \frac{kh}{2k+2} \right) + \\ &\quad \frac{(E_c \alpha_1 + E_1 \alpha_c) h \Delta T}{(k+1)(1-\mu)} \left(z_0 + \frac{kh}{2k+4} \right) \end{aligned}$$

由弹性理论可知, 功能梯度材料板在垂直于板面的外分布载荷 $q(x, y)$ 作用下的内力应满足以下各式:

$$\begin{cases} \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} = Q_x \\ \frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} = Q_y \end{cases} \quad (8)$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} + q(x, y) = 0 \quad (9)$$

式中, N_x 、 N_y 、 N_{xy} 是由横向分布载荷 $q(x, y)$ 引起的中面拉力.

记中面内点沿 x (或 y) 方向的位移为 u (或 v), 得板中面内点的应变表达式为

$$\begin{cases} \epsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \\ \epsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \\ \gamma_{xy} = \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \end{cases} \quad (10)$$

由式(8),得相容方程

$$\frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \quad (11)$$

板中面上应力还可用板中面内点的应变表示为

$$\begin{cases} \sigma_x^0 = \frac{E(z)}{1-\mu^2} [\epsilon_x + \mu \epsilon_y - (1+\mu)\alpha(z)\Delta T] \\ \sigma_y^0 = \frac{E(z)}{1-\mu^2} [\epsilon_y + \mu \epsilon_x - (1+\mu)\alpha(z)\Delta T] \\ \tau_{xy}^0 = \frac{E(z)}{2(1+\mu)} \gamma_{xy} \end{cases} \quad (12)$$

由式(12)可求得板中面拉力为

$$\begin{cases} N_x = \frac{Eh}{1-\mu^2} (\epsilon_x + \mu \epsilon_y) - N_T \\ N_y = \frac{Eh}{1-\mu^2} (\epsilon_y + \mu \epsilon_x) - N_T \\ N_{xy} = \frac{Eh}{2(1+\mu)} \gamma_{xy} \end{cases} \quad (13)$$

$$\text{式中, } E = E_c + \frac{E_1}{k+1}, N_T = \frac{E_c \alpha_c h \Delta T}{1-\mu} + \frac{E_1 \alpha_1 h \Delta T}{(2k+1)(1-\mu)} + \frac{(E_1 \alpha_c + E_c \alpha_1) h \Delta T}{(k+1)(1-\mu)}.$$

由式(13)还可以得到板中面点应变的另一种表达式为

$$\begin{cases} \epsilon_x = \frac{1}{Eh} (N_x - \mu N_y) + \frac{(1-\mu)N_T}{Eh} \\ \epsilon_y = \frac{1}{Eh} (N_y - \mu N_x) + \frac{(1-\mu)N_T}{Eh} \\ \gamma_{xy} = \frac{2(1+\mu)}{Eh} N_{xy} \end{cases} \quad (14)$$

$$\text{再令} \quad N_x = \frac{\partial^2 \varphi}{\partial y^2}, N_y = \frac{\partial^2 \varphi}{\partial x^2}, N_{xy} = -\frac{\partial^2 \varphi}{\partial x \partial y} \quad (15)$$

把式(7)、式(8)、式(15)代入式(9)中,把式(14)、式(15)代入式(11)中即可得到功能梯度材料板在均匀温度场中非线性弯曲方程组为

$$\begin{cases} D \nabla^4 w + \nabla^2 M_T = \left(\frac{\partial^2 \varphi}{\partial x^2} \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 \varphi}{\partial y^2} \frac{\partial^2 w}{\partial x^2} - 2 \frac{\partial^2 \varphi}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} \right) + q(x, y) \\ \nabla^4 \varphi + (1-\mu) \nabla^2 N_T = Eh \left[\left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 - \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right] \end{cases} \quad (16)$$

$$\text{式中, } \nabla^4 = \frac{\partial^4}{\partial x^4} + 2 \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}, \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}.$$

若圆板在均匀温度场中发生轴对称非线性弯曲变形时,引入极坐标可把式(16)化为

$$\begin{cases} D \frac{d}{dr} (\nabla^2 w) + \frac{dM_T}{dr} = \frac{1}{r} \frac{d\varphi}{dr} \frac{dw}{dr} + \frac{1}{r} \int_0^r q r dr \\ \frac{d}{dr} (\nabla^2 \varphi) + (1-\mu) \frac{dN_T}{dr} = -\frac{Eh}{2r} \left(\frac{dw}{dr} \right)^2 \end{cases} \quad (17)$$

$$\text{式中, } \nabla^2 = \frac{d^2}{dr^2} + \frac{1}{r} \frac{d}{dr}$$

2 圆板非线性弯曲计算

如图 2 所示,均匀温度场中功能梯度材料圆板,均布载荷 q_0 ,坐标原点在圆板中心,周边固支,有

$$r=0, \frac{d\varphi}{dr}=0; r=a,$$

$$\frac{d^2 \varphi}{dr^2} - \frac{u}{r} \frac{d\varphi}{dr} + (1 - \mu) N_T = 0 \quad (18a)$$

$$r = a, w(a) = 0 \quad (18b)$$

参阅有关弹性力学专著可设图2所示功能梯度材料圆板大挠度函数满足式(18b)时为

$$w(r) = f \left(1 - \frac{r^2}{a^2}\right)^2 \quad (19)$$

式中, f 为圆板中心挠度。

把式(19)代入式(17)第二分式中可以得到

$$\frac{d\varphi}{dr} = \frac{Eh f^2}{6a} \left[\frac{(5-3\mu)r}{(1-\mu)a} - \frac{6r^3}{a^3} + \frac{4r^5}{a^5} - \frac{r^7}{a^7} \right] - N_{Tr} \quad (20)$$

利用伽辽金原理可把式(17)第一分式化为

$$\int_0^a \left[D \frac{d}{dr} (\nabla^2 w) + \frac{dM_r}{dr} - \frac{1}{r} \int_0^r q r dr - \frac{1}{r} \frac{d\varphi}{dr} \frac{dw}{dr} \right] \left(\frac{r}{a} - \frac{r^3}{a^3} \right) r dr = 0 \quad (21)$$

把式(19)、式(20)代入式(21)中可得圆板中心挠度与载荷关系式为

$$\frac{qa^4}{h^4} = \frac{2(23-9\mu)E}{21(1-\mu)} \left(\frac{f}{h} \right)^3 + \left(\frac{64D}{h^3} - \frac{4a^2 N_T}{h^3} \right) \left(\frac{f}{h} \right) \quad (22)$$

当 $\Delta T = 0$ 、 $\mu = 0.316$ 、 $k = 0$ 且 $\frac{f}{h}$ 分别取 1、1.2、1.4、1.6 时, 式(22)即为各项同性材料板的计算结果与文献[15]方法误差分别为 0.476 2 %、0.842 9 %、1.221 4 %、1.599 1 %, 二者误差较小, 验证了本文方法的可靠性。

3 算例分析

采用 ANSYS 程序和本文方法计算了图2所示圆板中点挠度 f , 并比较了直接考虑 $z_0 = 0$, 即认为中面与中性面重合的情况。圆板半径 $a = 1\,000$ mm, 板厚 $h = 100$ mm。陶瓷材料的弹性模量和热膨胀系数分别为 $E_c = 380$ GPa、 $\alpha_c = 7.4 \times 10^{-6}/^\circ\text{C}$, 金属材料的弹性模量和热膨胀系数分别为 $E_m = 70$ GPa、 $\alpha_m = 23 \times 10^{-6}/^\circ\text{C}$, 泊松比均为 $\mu = 0.3$ 。 k 分别取 0, 0.25, 0.5。有限元建立两种模型求解, $k = 0$ 时, 材料模型 *mat1*, $E = 70$ GPa, $\mu = 0.3$, $\alpha_m = 23 \times 10^{-6}/^\circ\text{C}$, 单元最大边长尺寸为 20 mm, 单元为 8 节点 SOLID185 单元; $k = 0.5$ 时, 单元为 8 节点 SOLID46 实体层状单元, 定义 50 层材料层来模拟功能梯度材料材料性能的变化, 顶层 $E_c = 380$ GPa, $\alpha_c = 7.4 \times 10^{-6}/^\circ\text{C}$, $\mu = 0.3$ 底层 $E_m = 70$ GPa, $\alpha_m = 23 \times 10^{-6}/^\circ\text{C}$, $\mu = 0.3$, 中间层按照式(1)、式(2)来确定 E 和 α 。采用 Large Displacement static analysis 进行求解。 $k = 0, \Delta T = 20^\circ$, $qa^4/h^4 = 300$ GPa 和 $k = 0.5, \Delta T = 20^\circ$, $qa^4/h^4 = 300$ GPa 时圆板节点平面外位移如图3—图4所示, 本文方法与有限元程序结果比较如表1—表3所示。

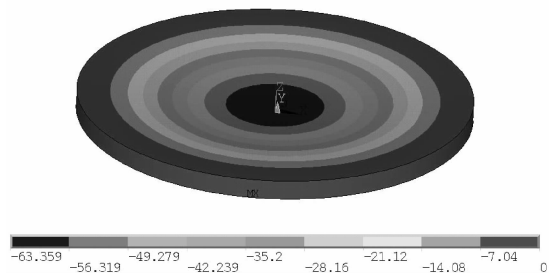


图3 节点平面外位移($k=0$)

Fig. 3 The nodes displacement out of plane

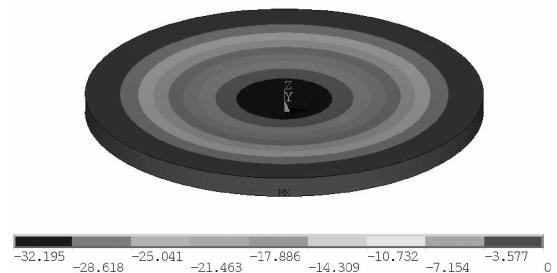


图4 节点平面外位移($k=0.5$)

Fig. 4 The nodes displacement out of plane

由表1—表3的计算结果分析可知,本文方法与有限元程序计算结果符合较好,验证了本文方法的可靠性;功能梯度材料圆板中点挠度随荷载增加而增加,呈现非线性特点;中点挠度随温度增加而增加,随 k 的增大而减小,但增大趋势随 k 的增大而减小.从算例分析表明,按照本文方法确定出的中性面位置计算出的结果和假设中性面在中面位置计算出的结果差异较大,差异最大可达15%,差异随着荷载的增加而增加,与温度变化无明显联系.由此可见中性面位置对温度场中功能梯度材料圆板的非线性弯曲有很大影响,认为中性面与中面重合将使结果产生较大的误差.

4 结 语

(1)本文方法与有限元程序计算结果符合较好,验证了本文方法的可靠性.

(2)均匀温度场中功能梯度材料圆板大挠度弯曲变形,中点挠度随温度增加而增加,随 k 的增大而减小,但增大趋势随 k 的增大而减小.

(3)中性面位置对温度场中功能梯度材料圆板的非线性弯曲有一定影响.

(4)假设中面位置在中面位置计算出的结果在荷载较小时误差较小,但随着荷载增加误差增加很快,在荷载较大时宜采用本文方法考虑中性面位置对计算结果的影响.

表1 载荷与挠度的非线性关系($\times 10^{10}$)($k=0$)

Tab.1 The nonlinear relation between load and deflection ($\times 10^{10}$)

$qa^4/h^4\ (\times 10^{10})$		5	10	20	30	40	50	
$\Delta T = 20^\circ$	f	This paper	12.75	24.98	46.22	63.88	78.59	90.82
		ANSYS	12.53	24.56	45.87	63.36	77.83	90.01
$\Delta T = 40^\circ$	f	This paper	13.27	25.91	48.01	65.67	80.35	92.78
		ANSYS	13.11	25.72	47.62	65.12	79.71	91.82
$\Delta T = 80^\circ$	f	This paper	14.68	28.50	51.58	69.63	84.37	96.77
		ANSYS	14.50	28.21	51.09	68.76	83.65	95.33

表2 载荷与挠度的非线性关系($\times 10^{10}$)($k=0.25$)

Tab.2 The nonlinear relation between load and deflection ($\times 10^{10}$)

$qa^4/h^4\ (\times 10^{10})$		5	10	20	30	40	50	
$\Delta T = 20^\circ$	f	$z = 0.1h$	6.94	13.78	26.85	38.85	49.52	59.11
		$z = 0$	6.09	12.13	23.79	34.73	44.82	53.99
		ANSYS	6.75	13.45	26.35	38.17	48.43	58.03
$\Delta T = 40^\circ$	f	$z = 0.1h$	7.23	14.35	27.89	40.13	50.93	60.68
		$z = 0$	6.31	12.59	24.59	35.88	46.08	55.35
		ANSYS	7.11	14.15	27.31	39.35	50.02	59.66
$\Delta T = 80^\circ$	f	$z = 0.1h$	7.86	15.65	30.03	42.86	54.08	63.95
		$z = 0$	6.79	13.58	26.37	38.15	48.75	58.28
		ANSYS	7.63	15.10	29.33	41.92	52.86	62.55

表3 载荷与挠度的非线性关系($\times 10^{10}$)($k=0.5$)

Tab.3 The nonlinear relation between load and deflection ($\times 10^{10}$)

$qa^4/h^4\ (\times 10^{10})$		5	10	20	30	40	50	
$\Delta T = 20^\circ$	f	$z = 0.12h$	5.67	11.30	22.15	32.29	41.57	50.03
		$z = 0$	4.73	9.47	18.76	27.58	35.88	43.68
		ANSYS	5.59	11.24	22.08	32.19	41.45	49.33
$\Delta T = 40^\circ$	f	$z = 0.12h$	5.88	11.71	22.93	33.31	42.78	51.36
		$z = 0$	4.90	9.76	19.33	28.35	36.87	44.79
		ANSYS	5.80	11.60	22.81	33.19	42.60	51.18
$\Delta T = 80^\circ$	f	$z = 0.12h$	6.37	12.72	24.59	35.56	45.41	54.22
		$z = 0$	5.23	10.43	20.56	30.51	38.96	47.35
		ANSYS	6.30	12.63	24.33	34.15	44.65	53

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Nonlinear bending calculation of FGM circular plate in the temperature field

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Abstract: Static equilibrium equation of FGM circular plate was established by using elastic mechanics theory. The location of neutral plane in FGM circular plate was determined by the utilization of static equilibrium equation. The nonlinear bending deformation differential equations of FGM circular plate in temperature field was derived, and the deflection function of FGM circular plate acquired. The effects of temperature, index of gradient on nonlinear bending deformation of FGM circular plate were discussed. Through FEM analysis of elliptical plate, the correctness of the method was verified. The main conclusions are as follows: the position of neutral surface was influential to nonlinear bending deformation of FGM circular plate and should consider the impact of the location of the neutral plane of bending when loading was large enough.

Key words: temperature field; functionally graded material (FGM); circular plate; nonlinear; bending deformation