基于能量原理的肋型板等效刚度

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摘 要:应用刚度等效原理导出肋型板的等代无肋板厚度计算显式,方法既适用于不同纵、横梁布置的板,也适用于板边各种支承情况,包括边界点支承的板. 针对工程中肋型板不同的边界约束状况,给出挠曲试函数各种形式. 计算过程涉及的数值积分简便易行,便于工程技术人员运用,与有限单元法相比,无需计算机程序. 多个典型算例的结果表明,代换前后板上的挠度分布特征及扭转角分布特征与有限元法计算结果在 3%~6% 误差范围一致.

关键词:现浇肋型板;等效刚度;能量法

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肋型板包括各种边界支承条件下的楼(屋)面梁板结构、井字楼盖. 肋型板竖向刚度是设计中关注的问题之一. 由于竖向荷载作用下梁肋和板协同工作,板的刚度、梁肋刚度及板边支承条件的差异均会对这一结构体系的组合刚度产生影响,从而对肋板作用效应产生影响. 对于肋型板的作用效应,常规的计算用有限单元法完成[1-4],相对于无肋板而言,可供参考的计算结果并无现成的显式表述. 我们从变形能等效原理出发,导出将各种肋形板等效为无肋板的等效刚度计算显式.

1 等变形能刚度代换

理论推理过程的基本假设是:具有相同平面尺寸 $a\times b$ 的肋型板(图 1a)与无肋板(图 1b)在相同荷载 q 作用下具有相同的弹性变型能,根据虚功原理,a 状态外力所做的功恒等于肋板变形后板内积畜的变形能 U_a :

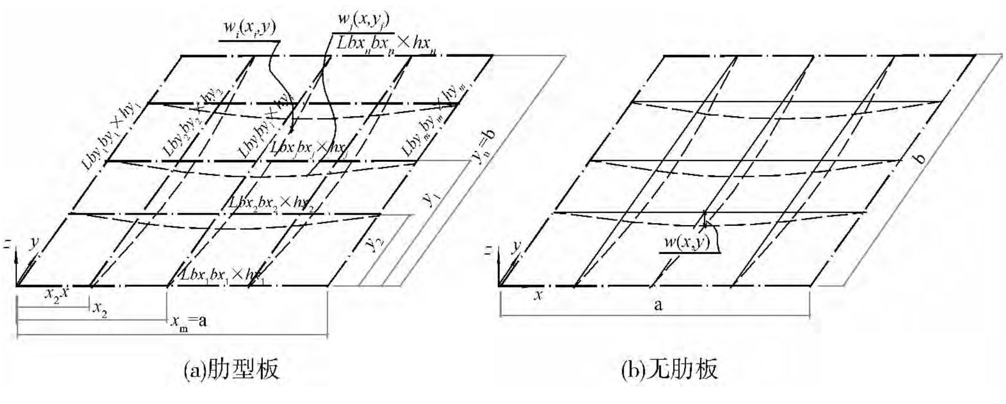

图 1 等刚度肋型板及无肋型板的变形记

Fig. 1 Deformation markers of equivalent to rib slab and unribbed slab

$$\iint_{(A)} qW_a(x,y)\mathrm{d}x\mathrm{d}y = U_a \tag{a}$$

同理,b 状态外力 q 所做的功恒等于肋板变形后板内积畜的变形能 U_b

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$$\iint_{(A)} qW_b(x,y)\mathrm{d}x\mathrm{d}y = U_b \tag{b}$$

根据基本假设:$U_a = U_b$,由(a)、(b)式有

$$\iint_{(A)} q(x,y)W_a(x,y)\mathrm{d}x\mathrm{d}y = \iint_{(A)} q(x,y)W_b(x,y)\mathrm{d}x\mathrm{d}y \tag{c}$$

因此可设 a、b 两状态的板有相同的挠曲试函数 $w_a(x,y) = w_b(x,y) = w(x,y)$,即肋板与无肋板刚度相同. 基于这一等刚度原则,a 状态的变形能由板和 x 向梁肋 $BL_{xj}(j = 1,2,\cdots,n)$ 及 y 向梁肋 $BL_{yi}(i = 1,2,\cdots,m)$ 共同贡献:

$$U = \frac{D_a}{2}\int_0^b\int_0^a(\nabla^2 w)^2\mathrm{d}x\mathrm{d}y + \frac{1}{2}\sum_{j=0}^{n}\int_0^a EI_{xj}(\,)\mathrm{d}x + \frac{1}{2}\sum_{i=0}^{m}\int_0^a EI_{yi}(\,)\mathrm{d}y +$$

$$\frac{1}{2}\sum_{j=0}^{n}\int_0^a GI_{Pxj}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2_{y=y_j}\mathrm{d}x + \frac{1}{2}\sum_{i=0}^{m}\int_0^a GI_{Pyi}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2_{x=x_i}\mathrm{d}y \tag{1}$$

其中:第一项面积分为周边简支或周边固支板的弹性变形能,第二、三项线积分分别为 x 向梁肋及 y 向梁肋的弯曲变形能,最后两项积分分别为 x 向梁肋及 y 向梁肋的扭转变形能. 由于梁肋与板变形的协调性,梁肋的挠曲函数均由板的挠曲试函数 $w(x,y)$ 唯一确定:

x 向梁肋挠曲函数为:$w(x,y_j)(j=1,2,\cdots,n)$,因此其曲率 $\kappa_{x_j} = \dfrac{\partial^2 w(x,y_j)}{\partial x^2}$,其单位杆长的扭转角 $\theta_{x_j} = \left(\dfrac{\partial^2 w}{\partial x\partial y}\right)_{y=y_j}$,同理,$y$ 向梁肋扭转角 $\theta_{y_i} = \left(\dfrac{\partial^2 w}{\partial x\partial y}\right)_{x=x_i}$. (1)式中 $D_a = \dfrac{E_a t_a^3}{12(1-\mu^2)}$ 为板的抗弯刚度,$EI_{xj} = \dfrac{E_a b_{xj} h_{xj}^3}{12}$,$EI_{yi} = \dfrac{E_a b_{yi} h_{yi}^3}{12}$ 分别为 x 向第 j 根梁肋抗弯刚度及 y 向第 i 根梁肋抗弯刚度,GI_{Pxj},GI_{Pyi} 分别为 x 向第 j 根梁肋抗扭刚度及 y 向第 i 根梁肋抗扭刚度.

对于无梁肋的 b 状态,其弹性变形能

$$U = \frac{D_b}{2}\int_0^b\int_0^a(\nabla^2 w)^2\mathrm{d}x\mathrm{d}y \tag{2}$$

$D_b = \dfrac{E_b t_b^3}{12(1-\mu^2)}$ 为无肋板的抗弯刚度.

根据基本假设,令(1)(2)式相等,可解得与肋型板(图 1a)等横向刚度的等厚度无肋板的厚度:

$$t_b = \left(\frac{E_a}{E_b}\right)^{\frac{1}{3}}\left\{t_a^3 + \left[\frac{U_M + U_T}{\int_0^b\int_0^a(\nabla^2 w)^2\mathrm{d}x\mathrm{d}y}\right]\right\}^{\frac{1}{3}} \tag{3}$$

其中 U_M 和 U_T 分别为梁肋的弯曲变形能及扭转变形能:

$$U_M = (1-\mu^2)\left[\sum_{j=1}^{n} b_{xj}h_{xj}^3\int_0^a\left(\frac{\partial^2 w_j}{\partial x^2}\right)^2\mathrm{d}x + \sum_{i=1}^{m} b_{yi}h_{yi}^3\int_0^b\left(\frac{\partial^2 w_i}{\partial y^2}\right)^2\mathrm{d}y\right] \tag{3a}$$

$$U_T = 6(1-\mu)\left[\sum_{j=0}^{n}\beta_{x_j}b_{x_j}^3 h_{x_j}\int_0^a\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2_{y=y_j}\mathrm{d}x + \sum_{i=0}^{m}\beta_{yi}b_{yi}^3 h_{yi}\int_0^b\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2_{x=xi}\mathrm{d}y\right] \tag{3b}$$

(3)式为本文导出的与肋型板(图 1a)具有相等横向刚度的等厚度板(图 1b)的厚度 t_b. t_b 在设定板的挠曲试函数 $w(x,y)$ 后即可求得,所涉及的积分一般均为简单初等函数的

定积分,运算过程并不冗繁. 当板周边非完全简支或完全固支时,第一项积分的被积函数 $(\nabla^2 w)^2$ 应改为[5]:$(\nabla^2 w)^2 - 2(1-\mu)\left[\dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2} - \left(\dfrac{\partial^2 w}{\partial x\partial y}\right)^2\right]$.

2　算　例

算例一:周边简支的井字楼盖(图 2)

以图 2 的井字楼盖为例,板平面尺寸 $a \times b = 12\ \mathrm{m} \times 16\ \mathrm{m}$,$x$ 向及 y 向等分数 $m=n=4$,梁肋截面尺寸 $b_{xj} = b_{yi} = 0.2\ \mathrm{m}$,$h_{xj} = h_{yi} = 0.4\ \mathrm{m}$,肋板厚 $t_a = 0.1\ \mathrm{m}$,$\mu = 0.15$,取挠曲试函数 $w(x,y) =$

$A\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}$，由(3)式计算等刚度代换后的无肋板厚 t_b

$$t_b=\left(\frac{E_a}{E_b}\right)^{\frac{1}{3}}\{t_a^3+t_M^3+t_T^3\}^{\frac{1}{3}} \tag{5}$$

$$t_M^3=\frac{2(1-\mu^2)}{(a^2+b^2)^2}\left[b^3\sum_{j=1}^{n}b_{xj}h_{xj}^3\ \sin^2\frac{j\pi}{n}+a^3\sum_{i=1}^{m}b_{yi}h_{yi}^3\ \sin^2\frac{i\pi}{m}\right] \tag{5a}$$

$$t_T^3=\frac{6(1-\mu)}{(a^2+b^2)^2}\left[2a^2b\sum_{j=0}^{n}\beta_{xj}b_{xj}^3h_{xj}\ \cos^2\frac{j\pi}{n}+2ab^2\sum_{i=0}^{m}\beta_{yi}b_{yi}^3h_{yi}\ \cos^2\frac{i\pi}{m}\right] \tag{5b}$$

将具体数值(5)式计算得等刚度代换后的无肋板厚 $t_b=0.156$ m.

图2给出该算例的有限元计算结果，其中无括号数度为肋形板变形数据，括号内数度为等刚度无肋板(周边支承梁肋仍保留)变形数据.

计算结果表明，将肋形板按等刚度转换的无肋板，其挠度 w 及转角 θ_x，θ_y 均较原肋板有所降低，码差在3%～8%范围.

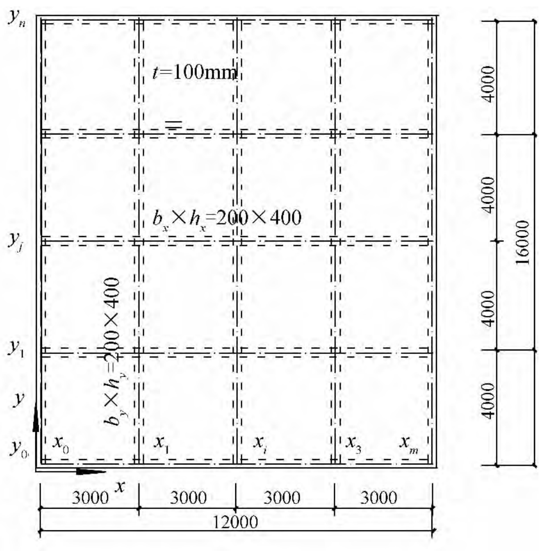
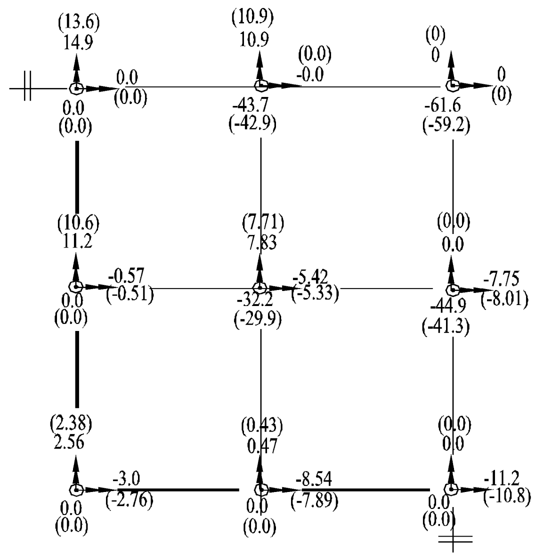


图2 周边简支井字楼盖几何参数(单位：mm)

Fig. 2 Geometrical parameters of the surrounding simply support cross-beam floor slab(unit：mm)



注：图中结果数据 $\times10^{-3}$ 挠度：m，转角：弧度

图3 左图边界简支肋板及等刚度无肋板变形有限元计算结果

Fig. 3 The calculation results of deformations of periphery surroundirg simply ribbed slab and the unribbed slab with equivalent-stiffress by finite element method (left figure)

算例二：周边固支的井字楼盖(图2)

取振型函数 $W(x,y)=A\left(1-\cos\frac{2\pi x}{a}\right)\left(1-\cos\frac{2\pi y}{b}\right)$，由(3)式计算等刚度代换后的无肋板厚 t_b 仍同(5)式

$$t_M^3=\frac{(1-\mu^2)}{\left[\frac{3a}{4b^3}+\frac{3b}{4a^3}+\frac{1}{2ab}\right]}\left[\frac{1}{2a^3}\sum_{j=1}^{n}b_{xj}h_{xj}^3\ \left(1-\cos\frac{2j\pi}{n}\right)^2+\frac{1}{2b^3}\sum_{i=1}^{m}b_{yi}h_{yi}^3\ \left(1-\cos\frac{2i\pi}{m}\right)^2\right] \tag{6a}$$

$$t_T^3=\frac{6(1-\mu)}{\left[\frac{3a}{4b^3}+\frac{3b}{4a^3}+\frac{1}{2ab}\right]}\left[\frac{1}{2ab^2}\sum_{j=1}^{n}\beta_{xj}b_{xj}^3h_{xj}\ \sin^2\frac{2j\pi}{n}+\frac{1}{2a^2b}\sum_{i=1}^{m}\beta_{yi}b_{yi}^3h_{yi}\ \sin^2\frac{2i\pi}{m}\right] \tag{6b}$$

等刚度代换后的无肋板厚 $t_b=0.12$ m.

算例三：周边简支的肋型楼盖(图4)

在图 4 中,肋板平面尺寸 $a\times b=12\ \mathrm{m}\times 16\ \mathrm{m}$,y 向等分数 $n=2$,x 向梁等分数 $m=4$,y 向梁肋截面尺寸 $b_{yi}=0.2\ \mathrm{m}$, $h_{yi}=0.4\ \mathrm{m}$,x 向梁肋截面尺寸 $b_{xj}=0.3\ \mathrm{m}$, $h_{xj}=0.8\ \mathrm{m}$,肋板厚 $t_a=0.1\ \mathrm{m}$, $\mu=0.15$,由(5a)(5b)式计算得等刚度代换后的无肋板厚 $t_b=0.24\ \mathrm{m}$

算例四:周边固支的肋型楼盖(图 4)若上例中的肋型板周边固支,将相应数据代入(6a)(6b)式后,由(5)式计算得等刚度代换后的无肋板厚 $t_b=0.15\ \mathrm{m}$

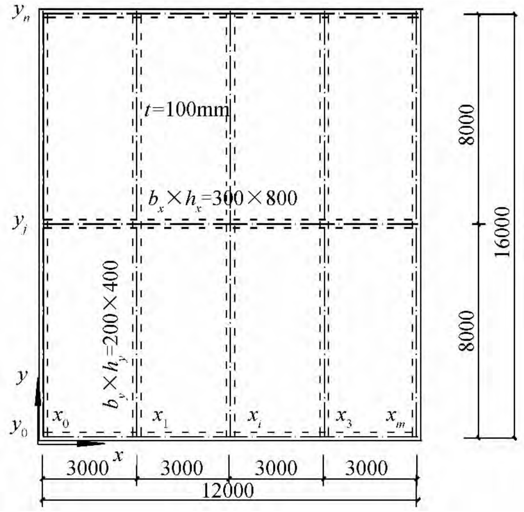


图 4　肋型板几何参数(单位:mm)

Fig. 4　Geometrical parameters of gird-beam-slab(unit:mm)

3　误差分析

大量计算结果表明,按本文方法计算的等刚度无肋板厚度偏差 3%～6%. 是典型的. 初步分析认为,引起这一误差的原因之一是(3)式分子中只考虑了肋板的弯曲变形能和扭转变形能,其剪切变形能未计入,原因之二是实际肋板的变形形态要比所设试函数 $w(x,y)=f_1(x)f_2(y)$ 更为复杂,因此按(3a)(3b)式计算的弯扭变形能存在偏差. 以上二因素引起(3)计算得的等代无肋板厚 t_b 存在偏差. 但这一误差尚在工程可接受范围.

4　板的挠曲试函数 $w(x,y)$

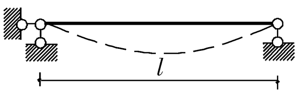
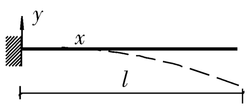
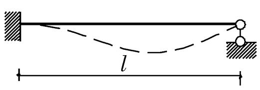
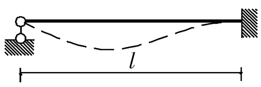
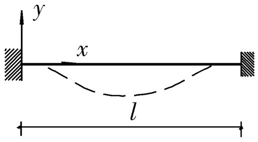
应用(3)式分析计算无肋板厚度 t_b 的关键是正确设定板的挠曲试函数 $w(x,y)$,$w(x,y)$ 可用分离变量的初等连续函数 $f_1(x)$,$f_2(y)$ 拼凑得到[1]:

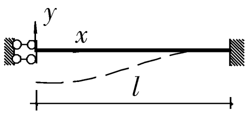
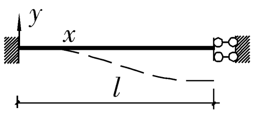
$$w(x,y)=Af_1(x)f_2(y) \tag{7}$$

$f_1(x)$,$f_2(y)$ 可采用表 1 给定的试函数形式,以满足不同板边约束条件.

表 1　挠曲试函数

Tab. 1　Deflection trial function

序号	振型曲线	振型函数 $f_1(x)$ 或 $f_2(y)$)
1		(1) $Y(x)=A\sin\frac{\pi x}{l}(0\leqslant x\leqslant l)$ (2) $Y(x)=A(x^4-2lx^3+l^3x)(0\leqslant x\leqslant l)$ (3) $Y(x)=A(3l^2x-4x^3)(0\leqslant x\leqslant l/2)$
2		(1) $Y(x)=A(1-\cos\frac{\pi x}{2l})(0\leqslant x\leqslant l)$ (2) $Y(x)=Ax^2(3l-x)(0\leqslant x\leqslant l)$ (3) $Y(x)=A\left(\frac{x}{l}\right)^2(0\leqslant x\leqslant l)$
3		$Y(x)=Ax^2\left(1-\frac{x}{l}\right)(0\leqslant x\leqslant l)$
4		$Y(x)=Ax\left(1-2\frac{x}{l}+\left(\frac{x}{l}\right)^2\right)(0\leqslant x\leqslant l)$
5		(1) $Y(x)=Ax^2(l-x)^2(0\leqslant x\leqslant l)$ (2) $Y(x)=A(1-\cos\frac{2\pi x}{l})(0\leqslant x\leqslant l)$ (3) $Y(x)=Ax^2\left(1-2\frac{x}{l}+\left(\frac{x}{l}\right)^2\right)(0\leqslant x\leqslant l)$

序号	振型曲线	振型函数 $f_1(x)$ 或 $f_2(y)$)
6		$Y(x)=A\left(1-3\left(\frac{x}{l}\right)^2+2\left(\frac{x}{l}\right)^3\right)(0\leqslant x\leqslant l)$
7		$Y(x)=A\left(3\left(\frac{x}{l}\right)^2-2\left(\frac{x}{l}\right)^3\right)(0\leqslant x\leqslant l)$

5 结 论

(1)应用能量原理导出肋型板与无肋板间等刚度转换的板厚度计算显式. 计算过程涉及的定积分运算简便,方法适用于工程技术人员应用. 一但与肋型板刚度等效的等厚无肋板厚度 t_b 求得后,则可应用弹性理论经典方法[5]或既有的相关表格[6-9]查出肋型板的设计控制挠度. 计算结果表明,按本文方法计算的无肋板厚度建模,有限元计算挠度分布、扭转角分布结果与等刚度肋形板在 3%—6%的误差范围一致.

(2)所得算式既适用于单向布肋的板,也适用于双向布肋的板,只需在计算式中改变纵、横向梁肋划分数 m,n.

(3)给出适于不同边界条件的挠曲试函数.

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Equivalent stiffness of rib board based on the energy principle

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Abstract: The method to derive the thickness calculation expression of rib board by the principle of equivalent stiffness of the rib board is applicable to not only the board arranged in transom and longitudinal beam, but also all kinds of support conditions at plate edges, including the supported plate on boundary points. In allusion to different types of boundary constraints status of rib board in engineering, various forms of buckling trial function are given. With only simple numerical integration involved in the calculation process, it is convenient for the application of engineering and technical personnel. Compared with finite element method, the computer program is not needed. The results of several typical examples show that the deflection distribution and reverse angle characteristics of the front and back plate are consistent with the calculation result of finite element method with the calculating error of 3%～6%.

Key words: *Cast-in-situ rib board*, *equivalent stiffness*, *the energy method*

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Beam Early Warning Method based on local stiffness detection and mode shape curvature

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Abstract: Highlighted in this paper is the damage detection and status early warning of beam structures with uncertain local flexural stiffness. Based on the concept of *Suppositional Partition*, static load is applied to the beam before serving and the beam should be simply supported. By using a novel loading system and mid-span deflection data processing method, actual local flexural stiffness value of each interval can be obtained by solving a set of linear equations. The obtained local stiffness data can be used to establish the finite element model of beams. Subsequently, mode identification is carried out for the beam in service and mode shape curvature index has been employed to detect the position of damage. Research results show that damage and original uncertainty of local flexural stiffness can be differentiated by this new method effectively and then the actual damage can be detected. Moreover, serviceability of the beam will not be affected.

Key words: *beam*; *early warning*; *stiffness*; *local*; *curvature*

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