

弹性半空间地基上正交异性矩形
薄板稳态振动通解王春玲^{1,2}, 张海霞¹, 丁 欢¹

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摘 要: 先对边界任意约束的正交各向异性矩形薄板, 构造了四次逐项可导的带有补充项的双重正弦傅里叶级数通解. 该解析解既不需要叠加, 对不同的物性参数又不需要分类, 而且待定系数少又具有明确的物理含义, 这使得正交各向异性矩形薄板的振动问题求解统一化、简单化、规律化. 然后将该通解与弹性半空间受任意竖向稳态荷载作用下的动力位移积分变换解相结合, 得出弹性半空间地基上边界任意约束的正交各向异性矩形板, 在任意竖向稳态荷载作用下的稳态振动解析解. 最后还给出了算例分析, 其结果与文献吻合良好, 证明本文的方法是切实可行的.

关键词: 弹性半空间地基; 正交各向异性矩形板; 相互作用; 稳态振动; 解析解

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许多工程实际问题都可抽象为搁置在弹性地基上的正交中异性矩形板问题. 如建筑物底层支柱的钢筋混凝土基础底板可视为该问题^[1]. 因此, 研究弹性地基上的正交各向异性矩形板稳态振动有重要的实际意义.

现有的研究, 大多数采用 Winkler 弹性地基模型或双参数弹性地基模型, 且大部分用数值方法或半解析方法研究. 文献[2]和文献[3]分别给出了弹性半空间地基上四边自由矩形板的弯曲、振动解析解, 但是均假定矩形板的材料是各向同性的, 文献[4]虽考虑了板材料的各向异性, 但仅限于弯曲问题. 而有关弹性半空间地基上正交各向异性矩形板稳态振动解析解国内外研究甚少.

本文将正交各向异性矩形薄板的带有补充项的双重正弦通解与弹性半空间受任意竖向稳态荷载作用下的动力位移积分变换解相结合, 得出弹性半空间地基上边界任意约束的正交各向异性矩形板, 在任意竖向稳态荷载作用下的稳态振动解析解.

1 控制微分方程及其边界条件

弹性半空间地基上正交异性矩形板($a \times b$), 受垂直于板面分布力 $q(x, y)e^{i\omega t}$ 作用, 若地基作用于板的竖向反力为 $F(x, y)e^{i\omega t}$, 取 x, y 轴与板的主方向平行, 若地基板的挠度为 $w e^{i\omega t}$, 则板的控制方程为:

$$D_x \frac{\partial^4 w}{\partial x^4} + 2H \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_y \frac{\partial^4 w}{\partial y^4} - \bar{m} \omega^2 w(x, y) = q(x, y) - F(x, y) \quad (1)$$

其中: \bar{m} 为板的面密度, D_x, D_y 分别为沿 x, y 方向的抗弯刚度, 它们同 x, y 方向的弹性模量 E_x, E_y , 泊松比 ν_x, ν_y 和板的厚度 h 等的关系, 见文献[5]. $H = D_x \nu_y + 2D_{xy}$ 为折算刚度, D_{xy} 为抗扭刚度. 板的内力幅值可以用挠度幅值函数表示为:

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$$M_x = -D_x \left(\frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} \right), M_{xy} = -2D_{xy} \frac{\partial^2 w}{\partial x \partial y}, V_x = -D_x \frac{\partial^3 w}{\partial x^3} - (H + 2D_{xy}) \frac{\partial^3 w}{\partial x \partial y^2}$$

以 $x = a$ 为例, 边界条件为: 简支边: $w = 0, M_x = 0$; 固定边: $w = 0, \frac{\partial w}{\partial x} = 0$;

自由边: $M_x = 0, V_x = 0$; 自由边的交点, 有角点条件: $\frac{\partial^2 w}{\partial x \partial y} = 0$.

2 板的一般解析通解

受文献[4]的启发, 我们提出以下四次逐项可导的带有补充项的双重正弦富氏级数解:

$$\begin{aligned} w = & \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} b_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} + \sum_{n=1}^{\infty} \left\{ \frac{1}{6a} \left[\frac{E_n - F_n}{D_x} + \frac{n^2 \pi^2 \nu_y (B_n - A_n)}{b^2} \right] x^3 + \frac{1}{2} \left[\frac{n^2 \pi^2 \nu_y A_n}{b^2} - \frac{E_n}{D_x} \right] x^2 \right. \\ & + \left[\frac{B_n - A_n}{a} + \frac{a}{6D_x} (F_n + 2E_n) - \frac{n^2 \pi^2 \nu_y (B_n + 2A_n)}{6b^2} \right] x + A_n \left. \right\} \sin \frac{n\pi y}{b} \\ & + \sum_{m=1}^{\infty} \left\{ \frac{1}{6b} \left[\frac{G_m - H_m}{D_y} + \frac{m^2 \pi^2 \nu_x (D_m - C_m)}{a^2} \right] y^3 + \frac{1}{2} \left[\frac{m^2 \pi^2 \nu_x C_m}{a^2} - \frac{G_m}{D_y} \right] y^2 \right. \\ & + \left[\frac{D_m - C_m}{b} + \frac{b}{6D_y} (H_m + 2G_m) - \frac{m^2 \pi^2 \nu_x (D_m + 2C_m)}{6a^2} \right] y + C_m \left. \right\} \sin \frac{m\pi x}{a} \\ & + \frac{1}{ab} (\omega_{00} + \omega_{ab} - \omega_{a0} - \omega_{0b}) xy + \frac{1}{a} (\omega_{a0} - \omega_{00}) x + \frac{1}{b} (\omega_{0b} - \omega_{00}) y + \omega_{00} \end{aligned} \quad (2)$$

显然, 当材料为各向同性时, 该解就退化成各向同性矩形板的通解. 这里, $\omega_{00}, \omega_{a0}, \omega_{0b}, \omega_{ab}$ 为板四个角点的挠度幅值, $A_n, B_n, E_n, F_n, C_m, D_m, G_m, H_m$ 分别为边界挠度及边界弯矩展开项的系数^[6]. 13 组待定系数, $\omega_{00}, \omega_{a0}, \omega_{0b}, \omega_{ab}, A_n, B_n, E_n, F_n, C_m, D_m, G_m, H_m$ 可用 8 个边界条件、四个角点条件及 1 个控制方程确定.

3 弹性半空间地基上四边自由正交各向异性矩形板稳态振动

3.1 建立定解方程

板的控制方程仍为(1), 边界条件为各边弯矩和总剪力同时等于零, 各角点反力为零. 将挠幅值度设成(2), 由各待定系数的物理意义及边界上弯矩为零的条件得: $E_n = F_n = G_m = H_m = 0$, 为了说明问题, 仅考虑载荷是对称的情况, 由对称性有, $A_n = B_n, C_m = D_m, \omega_{00} = \omega_{0b} = \omega_{a0} = \omega_{ab}$, 于是挠度表达式可简化为:

$$\begin{aligned} w = & \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} b_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} + \sum_{n=1}^{\infty} \left[\frac{n^2 \pi^2 \nu_y A_n}{2b^2} x^2 - \frac{n^2 \pi^2 \nu_y A_n}{2b^2} x + A_n \right] \sin \frac{n\pi y}{b} \\ & + \sum_{m=1}^{\infty} \left[\frac{m^2 \pi^2 \nu_x C_m}{2a^2} y^2 - \frac{m^2 \pi^2 \nu_x C_m}{2a^2} y + C_m \right] \sin \frac{m\pi x}{a} + \omega_{00} \end{aligned} \quad (3)$$

将荷载幅值及地基反力幅值均展为双重正弦级数:

$$q(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} q_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, F(x, y) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} Q_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \quad (4a, b)$$

将式(3), (4) 代入(1) 中, 将其中的多项式展为上述正弦级数, 并比较两边的系数可得:

$$\begin{aligned} & \{ [1 + (-1)^{n+1}] [D_x \left(\frac{m\pi}{a} \right)^4] \left(\frac{2}{n\pi} - \frac{2m^2 \pi^2 \nu_x b^2}{a^2 n^3 \pi^3} \right) - \frac{4H}{n\pi} \left(\frac{m\pi}{a} \right)^4 \nu_x [1 + (-1)^{n+1}] \} C_m \\ & + \{ [1 + (-1)^{m+1}] [D_y \left(\frac{n\pi}{b} \right)^4] \left(\frac{2}{m\pi} - \frac{2n^2 \pi^2 \nu_y a^2}{b^2 m^3 \pi^3} \right) - \frac{4H}{m\pi} \left(\frac{n\pi}{b} \right)^4 \nu_y [1 + (-1)^{m+1}] \} A_n \\ & + \{ D_x \left(\frac{m\pi}{a} \right)^4 + D_y \left(\frac{n\pi}{b} \right)^4 + 2H \left(\frac{m\pi}{a} \right)^2 \left(\frac{n\pi}{b} \right)^2 \} b_{mn} = q_{mn} - Q_{mn} \quad (m = 1, 2, 3 \cdots; n = 1, 2, 3 \cdots) \end{aligned} \quad (5)$$

考虑边界条件,由 $V_x|_{x=0} = 0$,可得:

$$\sum_{m=1}^{\infty} \left\{ D_x \left[\frac{m^2 \pi^2 v_x}{2a^2} a_n^{(2)} - \frac{m^2 \pi^2 b v_x}{2a^2} a_n^{(1)} + a_n^{(0)} \right] - (H + 2D_{xy}) v_x a_n^{(0)} \right\} \left(\frac{m\pi}{a} \right)^2 C_m - (H + 2D_{xy}) \left(\frac{a}{2} \right) \left(\frac{m\pi}{a} \right)^4 v_y A_n + \sum_{m=1}^{\infty} b_{nm} \left[D_y \left(\frac{m\pi}{a} \right)^2 + (H + 2D_{xy}) \left(\frac{m\pi}{b} \right)^2 \right] \left(\frac{m\pi}{a} \right) = 0 \quad (n = 1, 2, 3 \cdots) \quad (6)$$

式中: $a_n^{(0)}, a_n^{(1)}, a_n^{(2)}, a_n^{(3)}$ 分别为 $1, y, y^2, y^3$ 的傅里叶展式系数. 由 $V_y|_{y=0} = 0$,得:

$$\sum_{n=1}^{\infty} \left\{ D_y \left[\frac{n^2 \pi^2 v_y}{2b^2} b_m^{(2)} - \frac{n^2 \pi^2 a v_y}{2b^2} b_m^{(1)} + b_m^{(0)} \right] - (H + 2D_{xy}) v_y b_m^{(0)} \right\} \left(\frac{n\pi}{b} \right)^3 A_n - (H + 2D_{xy}) \left(\frac{b}{2} \right) \left(\frac{n\pi}{a} \right)^4 v_x C_m + \sum_{n=1}^{\infty} b_{nm} \left[D_y \left(\frac{n\pi}{b} \right)^2 + (H + 2D_{xy}) \left(\frac{n\pi}{a} \right)^2 \right] \left(\frac{n\pi}{b} \right) = 0 \quad (n = 1, 2, 3 \cdots) \quad (7)$$

这里, $b_m^{(0)}, b_m^{(1)}, b_m^{(2)}, b_m^{(3)}$ 分别为 $1, x, x^2, x^3$ 的傅里叶展开系数. 由 $\frac{\partial^2 W}{\partial x \partial y} = 0$ 可得:

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} b_{nm} \frac{mn\pi^2}{ab} - \sum_{n=1}^{\infty} \frac{av_y}{2} \left(\frac{n\pi}{b} \right)^3 A_n - \sum_{m=1}^{\infty} \frac{bv_x}{2} \left(\frac{m\pi}{a} \right)^3 C_m = 0 \quad (8)$$

与文献[7]类似,将地基表面位移幅值和板的挠度幅值展成同样形式的双重正弦级数,由矩形薄板的挠度幅值与弹性地基表面的竖向位移幅值相等,得以下变形协调方程:

$$-\frac{l^2}{\pi^2 \mu a b} \frac{mn\pi^2}{a^2 b^2} \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} Q_{pq} \eta_{pqmn} = b_{nm} + [1 + (-1)^{m+1}] \left(\frac{2}{m\pi} - \frac{2n^2 \pi^2 v_y a^2}{b^2 m^3 \pi^3} \right) A_n + [1 + (-1)^{n+1}] \left(\frac{2}{n\pi} - \frac{2m^2 \pi^2 v_x b^2}{a^2 n^3 \pi^3} \right) C_m + [1 + (-1)^{m+1}] [1 + (-1)^{n+1}] \frac{4}{mn\pi^2} \omega_{00} \quad (m = 1, 2, 3 \cdots; n = 1, 2, 3 \cdots) \quad (9)$$

其中, $l^2 = \rho \omega^2 / \mu$, ρ 是地基密度, μ 为地基土的拉梅系数. η_{pqmn} 详见文献[7].

方程(5)–(9)联立求解得待定系数 $b_{nm}, A_n, C_m, \omega_{00}, Q_{nm}$,进而可求得板的挠度幅值及其内力幅值.

3.2 数值算例

算例1:考虑一支承在弹性半空间地基表面,边长 $a = 4$ m,厚度 $h = 0.2$ m的四边自由弹性方薄板的稳态振动.假设板与地基之间为光滑接触.地基泊松比为0.25,密度 $\rho = 2000$ kg/m³,弹性模量为 $E_s = 50$ MPa.板的泊松比 ν_x 为0.167,板的密度为 $\rho = 2550$ kg/m³,弹性模量 $E_x = 30$ GPa, $E_x/E_y = 40$, $D_{xy} = E_y h^3 / 12$.在板上作用均布稳态激励荷载,其幅值为 $q = 100$ kPa,频率为 $f = 10$ Hz.

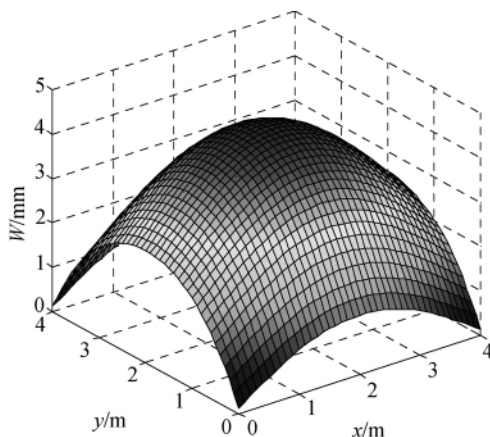


图1 弹性半空间地基板的挠度幅值

Fig. 1 Deflection amplitude of the plate

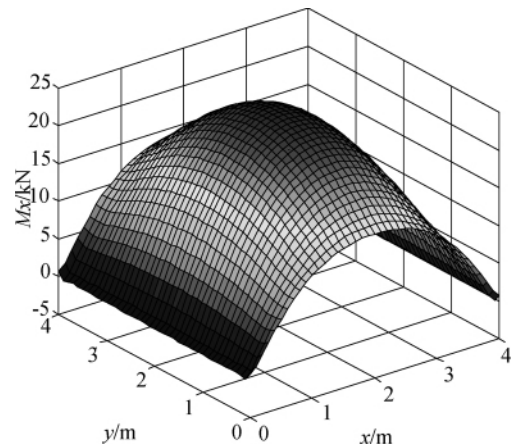


图2 弹性半空间地基板的弯矩 M_x 幅值

Fig. 2 Amplitude of moment M_x of the plate

计算时, m 和 n 都取到最大值 39, 其结果如图 1—图 2 所示. 该结果与文献[7] 的结果完全吻合.

4 弹性半空间地基上四边固定正交各向异性矩形板稳态振动

4.1 建立定解方程

此时板的控制微分方程仍为(1), 边界条件为在边界上挠度和转角均为零. 将挠度幅值仍设成(2)的形式, 由各待定系数的含义及边界上挠度为零的条件得: $A_n = B_n = C_m = D_m = 0$, $w_{00} = w_{a0} = w_{0b} = w_{ab} = 0$, 则挠度幅值函数表达式可简化为:

$$\begin{aligned} w = & \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} b_{nm} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} + \sum_{n=1}^{\infty} \left\{ \frac{E_n - F_n}{6aD_x} x^3 - \frac{E_n}{2D_x} x^2 + \frac{a}{6D_x} (F_n + 2E_n)x \right\} \sin \frac{n\pi y}{b} \\ & + \sum_{m=1}^{\infty} \left\{ \frac{G_m - H_m}{6bD_y} y^3 - \frac{G_m}{2D_y} y^2 + \frac{b}{6D_y} (H_m + 2G_m)y \right\} \sin \frac{m\pi x}{a} \end{aligned} \quad (10)$$

将(4)、(10) 式带入方程(1) 中, 将其中的多项式, 展为上述正弦级数, 类似分析可得:

$$\begin{aligned} (D_x \alpha_m^4 + D_y \beta_n^4 + 2H \alpha_m^2 \beta_n^2) b_{nm} + \{ D_y \beta_n^4 (b_m^{(1)} \frac{a}{3D_x} - b_m^{(2)} \frac{1}{2D_x} + b_m^{(3)} \frac{1}{6aD_x}) - 2H \beta_n^2 (b_m^{(1)} \frac{a}{aD_x} - b_m^{(0)} \frac{1}{D_x}) \} E_n \\ + \{ D_y \beta_n^4 (b_m^{(1)} \frac{a}{6D_x} - b_m^{(3)} \frac{1}{6aD_x}) + 2H \beta_n^2 \frac{b_m^{(1)}}{aD_x} \} F_n + \{ D_x \alpha_m^4 (a_n^{(1)} \frac{b}{3D_y} - a_n^{(2)} \frac{1}{2D_y} + a_n^{(3)} \frac{1}{6bD_y}) \\ - 2H \alpha_m^2 (a_n^{(1)} \frac{b}{bD_y} - a_n^{(0)} \frac{1}{D_y}) \} G_m + \{ D_x \alpha_m^4 (a_n^{(1)} \frac{b}{6D_y} - a_n^{(3)} \frac{1}{6bD_y}) + 2H \alpha_m^2 \frac{a_n^{(1)}}{bD_y} \} H_m = q_{nm} - Q_{nm} \\ (m = 1, 2, \dots; n = 1, 2, \dots) \end{aligned} \quad (11)$$

由各边界转角为零的条件类似可得:

$$\sum_{m=1}^{\infty} \frac{m\pi}{a} b_{nm} + \frac{a}{6D_x} (2E_n + F_n) + \frac{2b^2}{n^3 \pi^3 D_y} \sum_{m=1}^{\infty} [G_m + (-1)^{n+1} H_m] \frac{m\pi}{a} = 0 \quad (n = 1, 2, 3 \dots) \quad (12)$$

$$\sum_{m=1}^{\infty} (-1)^m \frac{m\pi}{a} b_{nm} - \frac{E_n + 2F_n}{6D_x} a + \frac{2b^2}{n^3 \pi^3 D_y} \sum_{m=1}^{\infty} (-1)^m [G_m + (-1)^{n+1} H_m] \frac{m\pi}{a} = 0 \\ (n = 1, 2, 3 \dots) \quad (13)$$

$$\sum_{n=1}^{\infty} \frac{n\pi}{b} b_{nm} + \frac{2a^2}{m^3 \pi^3 D_x} \sum_{n=1}^{\infty} [E_n + (-1)^{m+1} F_n] \frac{n\pi}{b} + \frac{b}{6D_y} (2G_m + H_m) = 0 \quad (m = 1, 2, 3 \dots) \quad (14)$$

$$\sum_{n=1}^{\infty} (-1)^n \frac{n\pi}{b} b_{nm} + \frac{2a^2}{m^3 \pi^3 D_x} \sum_{n=1}^{\infty} (-1)^n [E_n + (-1)^{m+1} F_n] \frac{n\pi}{b} - \frac{b}{6D_y} (G_m + 2H_m) = 0 \\ (m = 1, 2, 3 \dots) \quad (15)$$

由矩形薄板的挠度幅值与弹性地基表面的竖向位移幅值相等, 类似可得以下变形协调方程:

$$\begin{aligned} -\frac{l^2}{\pi^2 \mu a b} \frac{m n p q \pi^4}{a^2 b^2} \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} Q_{pq} \eta_{pqmn} = b_{nm} + \frac{2a^2}{m^3 \pi^3 D_x} E_n + (-1)^{m+1} \frac{2a^2}{m^3 \pi^3 D_x} F_n \\ + \frac{2b^2}{n^3 \pi^3 D_y} G_m + (-1)^{n+1} \frac{2b^2}{n^3 \pi^3 D_y} H_m \quad (m = 1, 2, 3 \dots; n = 1, 2, 3 \dots) \end{aligned} \quad (16)$$

方程(11)—(16) 联立求解, 可得各待定系数 b_{nm} , E_n , F_n , G_m , H_m 及 Q_{nm} , 进而可求得板的挠度幅值及其内力幅值.

4.2 数值算例

算例 2: 考虑支承在弹性半空间上, 边长 $a = 4$ m, 厚度 $h = 0.2$ m 的四边固定弹性方薄板的稳态振动. 假设板与地基之间为光滑接触. 地基泊松比为 0.25, 密度 $\rho = 2\,000$ kg/m³, 弹性模量为 $E_s = 50$ MPa. 板的泊松比 $\nu_x = 0.3$, $\nu_y = 0.1$, 弹性模量 $E_x = 34\,300$ MPa, $D_{xy} = 0.2D_x$. 板上作用频率为 $f = 10$ Hz 的稳态均布荷载, 其幅值为 $q = 0.1$ MPa.

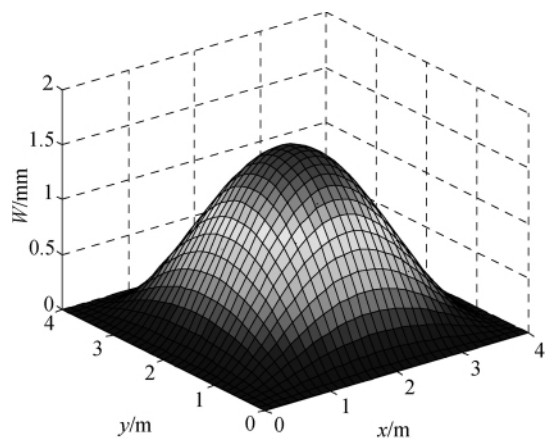
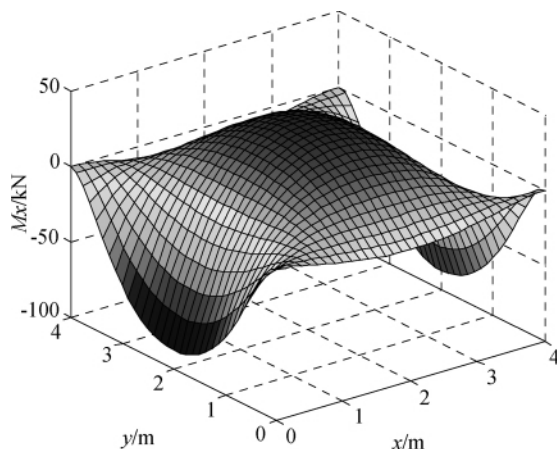


图 3 弹性半空间地基板的挠度幅值

Fig. 3 Deflection amplitude of the plate

图 4 弹性半空间地基板的弯矩 M_x 幅值Fig. 4 Amplitude of moment M_x of the plate

计算取 m, n 到 39, 所得结果如图 3—图 4 所示, 显然用本文的方法既不需要叠加, 且对不同的物性参数又不需要分类, 就可求弹性半空间地基上受任意边界约束的正交各向异性矩形板, 在任意竖向荷载作用下的稳态振动问题。

5 结 论

(1) 本文构造的四次逐项可导的带有补充项的双重正弦傅里叶级数通解, 可以求解边界任意约束的正交各向异性矩形薄板的弯曲、压弯、稳态振动及稳定问题。

(2) 该通解与弹性半空间受任意竖向简谐荷载作用下的动力位移积分变换解相结合, 可分析得出弹性半空间地基上边界任意约束的正交各向异性矩形板, 在任意竖向稳态荷载作用下的稳态振动解析解, 包括板的挠度和内力幅值。

(3) 本文的方法不仅具有普遍性而且弥补了数值法的一些不足, 同时取消了 Winkler 地基模型或双参数地基模型的假设, 从而得到板的内力幅值更合理、更精确的分布规律。

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General solutions to the steady vibration of orthotropic rectangular plates on the semi-infinite elastic foundation

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Abstract: General solution to the steady vibration which was constructed by double trigonometrically sine series with supplementary terms was introduced for the vibration analysis of orthotropic rectangular thin plates loaded with arbitrary steady vertical force on the semi-infinite elastic foundation. This general solution is four-order derivative, and has less undetermined coefficients which has definite physical meaning. The solution can be used to seth the problem of orthotropic rectangular plates on the semi-infinite elastic foundation with neither a dassification by different property parameter nor a superimposition. Such a solution has resulted in vibration analysis of orthotropic rectangular plates loaded with vertical steady force on the semi-infinite elastic foundation, unionization, simplification and systematization. The analytic vibration solution to orthotropic rectangular plates loaded with vertical steady force on the semi-infinite elastic foundation was given by combining the general solution of double trigonometrically sine series with supplementary terms with the dynamic integral representations for displacements of the semi-infinite elastic foundation loaded with arbitrary vertical steady force. The agreement is found satisfactory in proving the validity of the method in solving the problem. This new method would be feasible in practical applications.

Key words: *the semi-infinite elastic foundation; orthotropic rectangular plate; interaction; steady vibration; analytic solution*

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