不同模量面板夹心泡沫铝板大挠度弯曲

吴 晓，杨立军，黄 翀

(湖南文理学院土木建筑工程学院，湖南常德 415000)

摘要： 采用不同弹性理论研究了矩形不同模量面板夹心泡沫铝板大挠度弯曲问题，确定了矩形不同模量面板夹心泡沫铝板弯曲时的中性面位置．利用不同弹性理论建立了矩形不同模量面板夹心泡沫铝板在外载荷作用下的大挠度弯曲微分方程组，采用 Galerkin 原理求得了矩形不同模量面板夹心泡沫铝板中心挠度与均布载荷的关系式．通过算例分析讨论可知，当材料弹性模量相差较大时，不同模量弹性理论与经典弹性理论两种方法在矩形不同模量面板夹心泡沫铝板中心挠度的计算上存在较大的差异．

关键词： 不同模量，泡沫铝，层合板，大挠度，弯曲

中图分类号： O341　　**文献标志码：** A　　**文章编号：** 1006-7930(2014)02-0181-06

以夹心泡沫铝芯为代表的板夹层结构具有高强度比、高刚度比等优异特性[1]，在机械、航天航空等领域得到了广泛应用，所以夹心泡沫铝结构正成为研究发展的重点[2-6]．近年来国外研究机构已经研制成功了夹心泡沫铝结构，其结构面板可采用铝、铁、金属合金等材质，也可与泡沫铝芯实现冶金结合．当其为不同模量合金材料时[7-11]，即夹心泡沫铝板弯曲时，由于模量面板材料的拉压弹性模量有差异，不同模量面板夹心泡沫铝芯板则可以看作是三种不同材料组成的层合板．对不同模量弹性板的弯曲，国内外许多学者进行了研究，已建立了不同模量弹性理论[12-13]，并采用数值计算方法及半解析与数值结合的方法[14-15]研究不同模量结构的变形．因为解析法解是检验数值方法的计算精度重要手段，所以一般学者对采用解析法研究不同模量结构的重视程度不够．基于上述因素，本文采用不同模量弹性理论建立了不同模量面板夹心泡沫铝板的大挠度弯曲微分方程，研究了不同模量面板夹心泡沫铝板的大挠度弯曲变形问题．

1　板的弯曲微分方程

1.1　中性面位置的确定

不同模量面板夹心泡沫铝板弯曲时，会形成了各向同性弹性模量不同的受拉区和受压区．所以不同模量面板夹心泡沫铝板弯曲时的应力表达式为

$$\begin{cases}\sigma_x=-\dfrac{E_i z}{1-\mu_i^2}(\dfrac{\partial^2 w}{\partial x^2}+\mu_i\dfrac{\partial^2 w}{\partial y^2})\\ \sigma_y=-\dfrac{E_i z}{1-\mu_i^2}(\dfrac{\partial^2 w}{\partial y^2}+\mu_i\dfrac{\partial^2 w}{\partial x^2})\\ \tau_{xy}=-\dfrac{E_i z}{1+\mu_i}\dfrac{\partial^2 w}{\partial x\partial y}\end{cases}\tag{1}$$

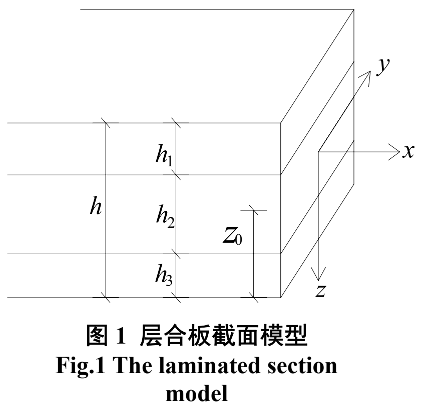

图 1 层合板截面模型

Fig.1 The laminated section model

式中，$i=1$ 时为受拉面板，$i=2$ 时为泡沫铝芯，$i=3$ 时为受压面板，如图 1 所示．E_1、E_3、E_2 分别为面板拉伸弹性模量、面板压缩弹性模量、泡沫铝芯弹性模量，μ_1、μ_3、μ_2 分别为面板拉伸泊松比、面板压缩泊松比、泡沫铝芯泊松比．

由不同模量弹性理论可得下式：

收稿日期： 2013-02-06　　**修改稿日期：** 2014-04-08

基金项目： 湖南省“十二五”重点建设学科(湘教发 2011[76])

作者简介： 吴晓(1965-)，男，教授，主要从事结构振动理论研究． E-mail:wx2005220@163.com

$$
\begin{cases}
\dfrac{E_1}{1-\mu_1^2}(\dfrac{\partial^2 w}{\partial x^2}+\mu_1\dfrac{\partial^2 w}{\partial y^2})\int_{z_0-h}^{z_0+h_1-h} z\,\mathrm{d}\,z+\dfrac{E_2}{1-\mu_2^2}(\dfrac{\partial^2 w}{\partial x^2}+\mu_2\dfrac{\partial^2 w}{\partial y^2})\int_{z_0+h_1-h}^{z_0-h_3} z\,\mathrm{d}\,z+ \\
\dfrac{E_3}{1-\mu_3^2}(\dfrac{\partial^2 w}{\partial x^2}+\mu_3\dfrac{\partial^2 w}{\partial y^2})\int_{z_0-h_3}^{z_0} z\,\mathrm{d}\,z=0 \\
\dfrac{E_1}{1-\mu_1^2}(\dfrac{\partial^2 w}{\partial y^2}+\mu_1\dfrac{\partial^2 w}{\partial x^2})\int_{z_0-h}^{z_0+h_1-h} z\,\mathrm{d}\,z+\dfrac{E_2}{1-\mu_2^2}(\dfrac{\partial^2 w}{\partial y^2}+\mu_2\dfrac{\partial^2 w}{\partial x^2})\int_{z_0+h_1-h}^{z_0-h_3} z\,\mathrm{d}\,z+ \\
\dfrac{E_3}{1-\mu_3^2}(\dfrac{\partial^2 w}{\partial y^2}+\mu_3\dfrac{\partial^2 w}{\partial x^2})\int_{z_0-h_3}^{z_0} z\,\mathrm{d}\,z=0
\end{cases}
\tag{2}
$$

利用式(2)可以得到:

$$
\frac{E_1}{1-\mu_1}\int_{z_0-h}^{z_0+h_1-h} z\,\mathrm{d}\,z+\frac{E_2}{1-\mu_2}\int_{z_0+h_1-h}^{z_0-h_3} z\,\mathrm{d}\,z+\frac{E_3}{1-\mu_3}\int_{z_0-h_3}^{z_0} z\,\mathrm{d}\,z=0 \tag{3}
$$

再利用式(3)可知中性面距不同模量面板夹心泡沫铝板下面板的距离即不同模量面板夹心泡沫铝板压缩区的高度为

$$
z_0=\frac{B_1h_1(2h_3+2h_2+h_1)+B_2h_2(2h_3+h_2)+B_3h_3^2}{2\left(B_1h_1+B_2h_2+B_3h_3\right)} \tag{4}
$$

式中: $B_1=\dfrac{E_1}{1-\mu_1}$, $B_2=\dfrac{E_2}{1-\mu_2}$, $B_3=\dfrac{E_3}{1-\mu_3}$.

1.2 大挠度弯曲方程

由式(1)可得到不同模量面板夹心泡沫铝芯板的弯矩及扭矩表达式

$$
\begin{aligned}
M_x=&-\frac{E_1}{1-\mu_1^2}(\frac{\partial^2 w}{\partial x^2}+\mu_1\frac{\partial^2 w}{\partial y^2})\int_{z_0-h}^{z_0+h_1-h} z^2\,\mathrm{d}\,z-\frac{E_2}{1-\mu_2^2}(\frac{\partial^2 w}{\partial x^2}+\mu_2\frac{\partial^2 w}{\partial y^2})\int_{z_0+h_1-h}^{z_0-h_3} z^2\,\mathrm{d}\,z- \\
&\frac{E_3}{1-\mu_3^2}(\frac{\partial^2 w}{\partial x^2}+\mu_3\frac{\partial^2 w}{\partial y^2})\int_{z_0-h_3}^{z_0} z^2\,\mathrm{d}\,z
\end{aligned}
\tag{5a}
$$

$$
\begin{aligned}
M_y=&-\frac{E_1}{1-\mu_1^2}(\frac{\partial^2 w}{\partial y^2}+\mu_1\frac{\partial^2 w}{\partial x^2})\int_{z_0-h}^{z_0+h_1-h} z^2\,\mathrm{d}\,z-\frac{E_2}{1-\mu_2^2}(\frac{\partial^2 w}{\partial y^2}+\mu_2\frac{\partial^2 w}{\partial x^2})\int_{z_0+h_1-h}^{z_0-h_3} z^2\,\mathrm{d}\,z- \\
&\frac{E_3}{1-\mu_3^2}(\frac{\partial^2 w}{\partial y^2}+\mu_3\frac{\partial^2 w}{\partial x^2})\int_{z_0-h_3}^{z_0} z^2\,\mathrm{d}\,z
\end{aligned}
\tag{5b}
$$

$$
M_{xy}=-\frac{E_1}{1+\mu_1}\frac{\partial^2 w}{\partial x\partial y}\int_{z_0-h}^{z_0+h_1-h} z^2\,\mathrm{d}\,z-\frac{E_2}{1+\mu_2}\frac{\partial^2 w}{\partial x\partial y}\int_{z_0+h_1-h}^{z_0-h_3} z^2\,\mathrm{d}\,z-\frac{E_3}{1+\mu_3}\frac{\partial^2 w}{\partial x\partial y}\int_{z_0-h_3}^{z_0} z^2\,\mathrm{d}\,z \tag{5c}
$$

根据不同模量弹性理论可知，当不同模量面板夹心泡沫铝板在载荷 $q(x,y)$ 作用下，载荷 $q(x,y)$ 与不同模量面板夹心泡沫铝板的内力满足以下式:

$$
\begin{cases}
\dfrac{\partial M_x}{\partial x}+\dfrac{\partial M_{xy}}{\partial y}=Q_x \\
\dfrac{\partial M_{xy}}{\partial x}+\dfrac{\partial M_y}{\partial y}=Q_y
\end{cases}
\tag{6}
$$

$$
\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+N_x\frac{\partial^2 w}{\partial x^2}+2N_{xy}\frac{\partial^2 w}{\partial x\partial y}+N_y\frac{\partial^2 w}{\partial y^2}+q(x,y)=0 \tag{7}
$$

式中： N_x、 N_y、 N_{xy} 是横向载荷 $q(x,y)$ 引起的中面拉力和剪力.

由不同模量弹性理论可知不同模量面板夹心泡沫铝板的中面应变可用位移表示为

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{1}{2}(\frac{\partial w}{\partial x})^2 \tag{8a}$$

$$\varepsilon_y = \frac{\partial v}{\partial y} + \frac{1}{2}(\frac{\partial w}{\partial y})^2 \tag{8b}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{8c}$$

利用式(8a)和(8c)可得不同模量面板夹心泡沫铝板中面的变形协调方程为

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} = (\frac{\partial^2 w}{\partial x \partial y})^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \tag{8d}$$

由于 N_x、 N_y、 N_{xy} 为横向载荷 $q(x,y)$ 引起的不同模量面板夹心泡沫铝芯板中面拉力和剪力，所以应变可用不同模量面板夹心泡沫铝板中面拉力和剪力表示为

$$\begin{cases} \varepsilon_x = \dfrac{1}{Eh}(N_x - \mu N_y) \\ \varepsilon_y = \dfrac{1}{Eh}(N_y - \mu N_x) \\ \gamma_{xy} = \dfrac{2(1+\mu)}{Eh} N_{xy} \end{cases} \tag{9}$$

式中：$E = \dfrac{E_1 h_1 + E_2 h_2 + E_3 h_3}{h}$，$\mu = \dfrac{\mu_1 h_1 + \mu_2 h_2 + \mu_3 h_3}{h}$．令 $N_x = h\sigma_x = h\dfrac{\partial^2 \varphi}{\partial y^2}$，$N_{xy} = h\tau_{xy} = -h\dfrac{\partial^2 \varphi}{\partial x \partial y}$　（10）

把式(5)、式(6)代入式(7)中，把式(9)、式(10)代入式(8)中，即可得到不同模量面板夹心泡沫铝板大挠度弯曲微分方程组

$$\begin{cases} D\nabla^4 w = h(\dfrac{\partial^2 \varphi}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2} + \dfrac{\partial^2 \varphi}{\partial y^2}\dfrac{\partial^2 w}{\partial x^2} - 2\dfrac{\partial^2 \varphi}{\partial x \partial y}\dfrac{\partial^2 w}{\partial x \partial y} + q(x,y) \\ \nabla^4 \varphi = E_1\left[\left(\dfrac{\partial^2 w}{\partial x \partial y}\right)^2 - \dfrac{\partial^2 w}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2}\right] \end{cases} \tag{11}$$

式中：　$\nabla^4 = \left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)^2$，

$$D = \frac{E_1}{3(1-\mu_1^2)}[(z_0 + h_1 - h)^3 - (z_0 - h)^3] + \frac{E_2}{3(1-\mu_2^2)}[(z_0 - h_3)^3 - (z_0 + h_1 - h)^3] + \frac{E_3}{3(1-\mu_3^2)}[z_0^3 - (z_0 - h_3)^3].$$

2　板的大挠度弯曲求解

设不同模量面板夹心泡沫铝板受到弹性肋条(图 2 中的虚线)的约束，并设沿短边 b 单位长度内平均分配的肋条面积为 A_x，沿长边 a 则为 A_y，层合板与肋条横截面面积之比分别为

$$\frac{h}{A_x} = \gamma_x, \quad \frac{h}{A_y} = \gamma_y \tag{12}$$

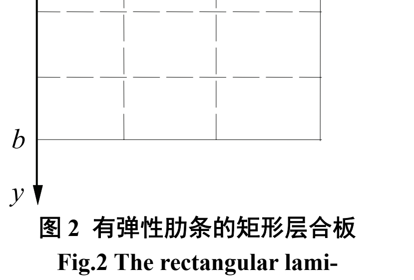

图 2　有弹性肋条的矩形层合板

Fig.2 The rectangular laminated plate with elastic grid

当不同模量面板夹心泡沫铝芯板边不能移动时，$A_x \to \infty$、$A_y \to \infty$；当不同模量面板夹心泡沫铝芯板边可以自由时，$A_x \to 0$、$A_y \to 0$．

假设不同模量面板夹心泡沫铝芯板四边为简支，可设其位移函数为

$$w(x,y)=f\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \tag{13}$$

把式(13)代入式(11)第二分式可得：

$$\varphi(x,y)=\frac{Ef^2}{32}[(\frac{a}{b})^2\cos\frac{2\pi x}{a}+(\frac{b}{a})^2\cos\frac{2\pi y}{b}]+\frac{1}{2}P_x y^2+\frac{1}{2}P_y x^2 \tag{14}$$

矩形不同模量面板夹心泡沫铝板的端部伸长为

$$\begin{cases}\Delta_x=-\int_0^a\int_0^b\left[\frac{1}{Eh}(\frac{\alpha^2\phi}{\alpha y^2}-\mu\frac{\alpha^2\phi}{\alpha x^2})-\frac{1}{2}(\frac{\alpha W}{\alpha x})^2\right]\mathrm{d}x\mathrm{d}y=\frac{a\nu_x P_x}{E}\\ \Delta_y=-\int_0^a\int_0^b\left[\frac{1}{Eh}(\frac{\alpha^2\phi}{\alpha x^2}-\mu\frac{\alpha^2\phi}{\alpha y^2})-\frac{1}{2}(\frac{\alpha W}{\alpha y})^2\right]\mathrm{d}x\mathrm{d}y=\frac{b\nu_y P_y}{E}\end{cases} \tag{15}$$

把式(13)、式(14)代入式(15)中可以得到：

$$\begin{cases}P_x=\frac{\pi^2Eh^2\xi^2}{8b^2}\frac{\mu+(1+\gamma_y)/\lambda^2}{(1+\gamma_x)(1+\gamma_y)-\mu^2}\\ P_y=\frac{\pi^2Eh^2\xi^2}{8b^2}\frac{\mu/\lambda^2+1+\gamma_x}{(1+\gamma_x)(1+\gamma_y)-\mu^2}\end{cases} \tag{16}$$

把式(13)、式(16)代入式(11)中第一分式中，利用伽辽金原理可得：

$$\frac{\pi^6}{128}[\frac{\mu/\lambda^2+1+\gamma_x}{(1+\gamma_x)(1+\gamma_y)-\mu^2}+\frac{\mu+(1+\gamma_y)/\lambda^2}{(1+\gamma_x)(1+\gamma_y)-\mu^2}\frac{1}{\lambda^2}+\frac{1}{2}(1+\frac{1}{\lambda^4})]\xi^3+\frac{\pi^6D}{16Eh^3}(1+\frac{1}{\lambda^2})^2\xi=q_0 \tag{17}$$

其中：$\lambda=\frac{a}{b},\xi=\frac{f}{h},q_0=\frac{q}{E}(\frac{b}{h})^4$.

当矩形不同模量面板夹心泡沫铝板为可移动简支时即 $\gamma_x\to\infty$、$\gamma_y\to\infty$，式(17)可以化为

$$\frac{\pi^6}{256}(1+\frac{1}{\lambda^4})\xi^3+\frac{\pi^6D}{16Eh^3}(1+\frac{1}{\lambda^2})^2\xi=q_0 \tag{18}$$

当矩形不同模量面板夹心泡沫铝芯板为不可移动简支时即 $\gamma_x=0$、$\gamma_y=0$，式(17)可以化

$$\frac{\pi^6}{128}[\frac{\lambda^4+2\mu\lambda^2+1}{(1-\mu^2)\lambda^4}+\frac{1}{2}(1+\frac{1}{\lambda^4})]\xi^3+\frac{\pi^6D}{16Eh^3}(1+\frac{1}{\lambda^2})^2\xi=q_0 \tag{19}$$

当 $E_1=E_2$，$\mu_1=\mu_2$ 时，式(18)、式(19)的表达式与经典弹性理论给出的计算公式一致的.

3　算例分析

为分析不同模量面板夹心泡沫铝板的大挠度弯曲，假设面板受拉时 E_1=66.93 GPa，μ_1=0.34，面板受压时 E_3=73.43 GPa，μ_3=0.39．泡沫铝芯 E_2=24 GPa，μ_2=0.34．为了验证本文计算方法正确性，分别用 ANSYS 和本文方法(即式(18)和式(19))计算了均布载荷作用下不同模量面板夹心泡沫铝板周边为可移动简支和不可移动简支的板中点挠度 f.矩形不同模量面板夹心泡沫铝板长边尺寸 a=2 000 mm 矩形板短边尺寸 b=1 000 mm，板厚 h=100 mm.模型由三层板组成,其中下层板厚 10 mm,材料模型 mat1，E_1=66.93 GPa，μ_1=0.34.中间夹心泡沫铝厚 80 mm,材料模型 mat2，E_2=24 GPa，μ_2=0.34，上层板板厚 10mm,材料模型 mat3，E_3=73.43 GPa，μ_3=0.39．单元最大边长尺寸 5 mm．单元为 8 节点 SOLID185 单元，采用 Large Displacement static analysis 进行求解．本文计算结果与有限元结果比较如表 1 所示．$qa^4/h^4=300$ GPa 时板节点平面外位移如图 3 和图 4 所示．本文计算结果与有限元结果比较如表 1 所示．

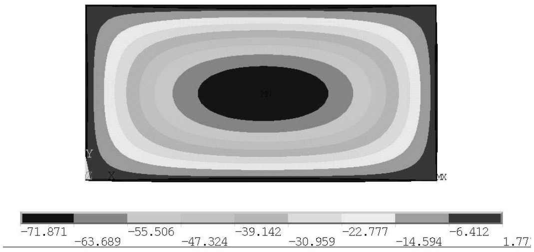

图 3 可移动简支板节点平面外位移
Fig.3 Out-plane displacement of moveable simply supported plate nodes

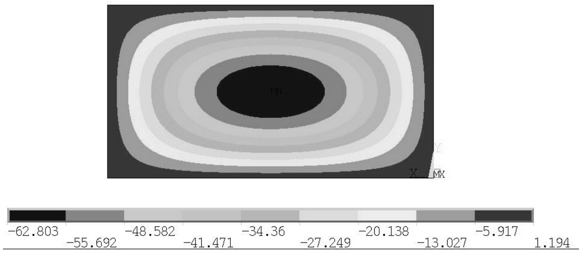

图 4 不可移动简支板节点平面外位移
Fig.4 Out-plane displacement of unmoveable simply supported circular plate nodes

表 1　本文结果与 ANSYS 结果比较
Tab.1 Comparison results between this paper and ANSYS

	qa^4/h^4 ($\times10^{10}$)	5	10	15	20	25	30
可移动简支	本文方法/mm	12.9	24.5	35.6	45.8	54.7	69.0
	ANSYS /mm	13.2	25.2	36.8	47.5	56.8	71.8
	误差/%	-2.3	-2.8	-3.2	-3.5	-3.7	-3.9
	$E_1=E_3$=66.93 GPa	14.7	28.6	41.5	52.6	62.8	80.1
	$E_1=E_3$=73.43 GPa	11.1	20.9	30.8	39.4	47.5	59.3
不可移动简支	本文方法/mm	11.7	22.5	32.0	40.3	47.5	59.9
	ANSYS /mm	12.1	23.3	33.2	41.8	49.3	62.8
	误差/%	-3.3	-3.4	-3.6	-3.6	-3.7	-4.6
	$E_1=E_3$=66.93 GPa	13.2	25.3	35.9	44.2	52.7	68.8
	$E_1=E_3$=73.43 GPa	10.5	19.2	27.5	34.7	41.1	51.9

由表 1 结果和图 3、图 4 可以看出，采用有限元方法研究矩形不同模量面板夹心泡沫铝板的大挠度弯曲和采用本文方法研究矩形不同模量面板夹心泡沫铝板的大挠度弯曲，两种方法的计算结果非常相近、吻合得较好，说明本文解析方法的计算结果是可靠的．对于矩形不同模量面板夹心泡沫铝板的大挠度弯曲计算，不考虑面板拉压弹性模量相异时其计算结果与实际情况相差较大，超过了工程上所允许的计算误差 5%．所以，对于面板拉压弹性模量相差较大的不同模量面板夹心泡沫铝板的大挠度弯曲计算，板中心挠度的计算不宜采用经典弹性理论，而应该采用不同模量弹性理论．

4　结论

由以上分析可以得到以下结论：

(1) 通过算例分析发现采用本文的解析方法计算矩形不同模量面板夹心泡沫铝板的大挠度弯曲和有限元方法计算的结果比较相近，误差均在 5 %以内．表明两种方法的计算结果吻合得较好，这说明本文解析方法的计算结果是可靠的．

(2) 算例分析表明，对于不同模量面板夹心泡沫铝板的大挠度弯曲计算，采用经典弹性理论的计算结果与采用不同模量弹性理论的计算结果相差较大，相对误差均在 30 %以上．

所以，对于面板拉压弹性模量相差较大的不同模量面板夹心泡沫铝板的大挠度弯曲计算，板中心挠度的计算不宜采用经典弹性理论，而应该采用不同模量弹性理论．

参考文献　References

[1] 王展光, 单建, 何德坪. 金字塔栅格夹心夹层板动力响应分析[J]. 力学季刊, 2006, 27(4):707-713.
WANG ZHANguang, SHAN Jian, HE Deping. Dynamic response analysis of sandwich plates with pyramidal truss cores[J]. Chinese Quarterly of Mechanics, 2006, 27(4):707-713.

[2] 于英华, 高华. 泡沫铝/铸铁层合梁弯曲性能有限元分析[J]. 现代制造工程, 2008(7): 39-42.
YU Yinghua, GAO Hua. Study on bending performance of foam aluminum/cast iron sandwich beam by FEA[J]. Modern Man-

ufacturing Engineering, 2008(7): 39-42.

[3] 梁晓军, 朱勇刚, 陈峰, 等. 泡沫铝芯三明治板的粉末冶金制备及其板/芯界面研究[J]. 材料科学与工程学报, 2005, 23(1): 77-80.
LIANG Xiaojun, ZHU Yonggang, CHEN Feng, et al. Research on preparation of aluminium foam sandwich by powder metallurgical method and face plate/foam core interfacial microstructure[J]. Journal of Materials Science and Engineering, 2005, 23(1): 77-80.

[4] 查海波, 凤仪, 朱琪琪, 等. 泡沫铝层合梁的弯曲性能[J]. 中国有色金属学报, 2007, 17(2):290-295.
ZHA Haibo, FENG Yi, ZHU Qiqi, et al. Bending capability of foam aluminum sandwich beams[J]. The Chinese Journal of Nonferrous Metals, 2007, 17(2):707-713.

[5] 尚金堂, 何德坪. 泡沫铝层合梁的三点弯曲变形[J]. 材料研究学报, 2003, 17(1):31-38.
SHANG Jintang, HE Deping. Deformation of sandwich beams with Al foam cores in three-point bending[J]. Chinese Journal of Materials Research, 2003, 17(1):31-38.

[6] 王青春, 范子杰, 桂良进, 等. 中等应变率下泡沫铝的吸能特性[J]. 材料研究学报, 2005, 19(6):601-607.
WANG Qingchun, FAN Zijie, GUI Liangjin, et al. Energy absorption behaviour of aluminium foam under medium strain rate[J]. Chinese Journal of Materials Research, 2005, 19(6):601-607.

[7] 李战莉，黄再兴．双模量泡沫材料等效弹性模量的细观力学估算方法[J]．南京航空航天大学学报，2006, 38(4): 464-468.
LI Zhanli, HUANG Zaixing. Meso-Mechanical method for estimating equivalent elastic modulus of foam-solid with double-modulus[J]. Journal of Nanjing University of Aeronautics & Astronautics, 2006, 38(4): 464-468.

[8] 曾纪杰．对中柔度压杆的双模量理论的修正[J]．机械强度，2006, 28(3): 462-464.
ZENG JiJie. Revision of the formula with bimodulusim intermediate column[J]. Journal of Mechanical Strength, 2006, 28(3): 462-464.

[9] 蔡来生，俞焕然．拉压模量不同弹性物质的本构[J]．西安科技大学学报，2009, 29(1): 17-21.
CAI Laisheng, YU Huanran. Constitutive relation of elastic materials with different elastic moduli in tension and compression[J]. Journal of Xi’an University of Science and Technology, 2009, 29(1): 17-21.

[10] 罗战友，夏建中，龚晓南．不同拉压模量及软化特性材料的柱形孔扩张问题的统一解[J]．工程力学，2008, 25(9): 79-84.
LUO Zhanyou, XIA Jianzhong, GONG Xiaonan. Unified solution for expansion of cylindrical cavity in strain-softening materials with different elastic moduli in tension and compression[J], Engineering Mechanics, 2008, 25(9): 79-84.

[11] 阿巴尔楚米扬．不同模量弹性理论[M]．邬瑞锋，张允真，译．北京：中国铁道出版社，1986:11-22.
Ambartsumyan S A，Elasticity theory of different modulus[M]．WU Ruifeng，ZHANG Yunzhen Translation． Beijing：China Railway Press，1986:11-22 .

[12] Meir Destrade. Biomodular rubber buckles early in bending[J].Mechanics of Materials,2009(42):469-476.

[13] Meir Barak. Are tensile and compressive Young′s moduli of compact bone different[J].Journal of the Mechanical Behavior of Bio-medical Materials, 2009(2): 51-60.

[14] Qu Chengzhong. Deformation of geocell with different tensile and compressive modulus[J]. Electronic Journal of Geotechnical Engineering,2009(14):1-14.

[15] Haitian Yanga, Mingling xuab. Solving inverse bimodular problems via artificial neural network[J]. Inverse Problems in Science and Engineering, 2009, 17(8): 999-1017.

Large deflection bending of different modulus panel aluminum foam core laminated plate

WU Xiao, *YANG Lijun*, *HUANG Chong*

(Hunan University of Arts and Science, Changde 415000, China)

Abstract: By using elasticity theory Large deflection problem of different modulus panel aluminum foam core laminated plate was studied. The location of neutral plane in panel aluminum foam core laminated plate was determined. The large deflection bending deformation differential equations of different modulus rectangular panel aluminum foam core laminated plate was derived, and the relation expression between central deflection of laminated plate and uniform load was obtained. The elastic theory on different modulus and the classical theory of elasticity of two kinds of method have great differences in the structural deflection calculation by calculation and analysis of examples.

Key words: different modulous; aluminum foam; laminated plate; large deflection; bending

（本文编辑　沈波）