

深埋隧道开挖引发的应力与位移非线性解

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摘要: 对隧道开挖引发的地层扰动进行应力和位移的快速及准确计算具有重要的理论与工程实际意义。本文基于非线性 Hoek-Brown 强度准则, 求解了隧道开挖后塑性区的应力和位移解析解, 并对应力和位移解进行了参数敏感性分析。研究表明: 塑性区半径随开挖半径的增大而增大; 当 $m_i \leq 20$ 时, r_p/r_0 随开挖半径的增大呈现非线性增大的演化规律, 当 $m_i = 25$, r_p/r_0 随着开挖半径的增大呈现先增后减趋势; 不同 m_i 值时, r_p/r_0 随着 GSI 增大而非线性增大; 剪胀和开挖半径对的影响中, 开挖半径对位移的影响更加显著。

关键词: 隧道开挖; Hoek-Brown 强度准则; 应力解; 位移解

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Nonlinear solution of stress and displacement induced by deep-buried tunnel excavation

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Abstract: The rapid and accurate calculation of stress and displacement of stratum disturbance caused by tunnel excavation has important theoretical and practical significance. Based on the nonlinear Hoek Brown strength criterion, the analytical solutions of stress and displacement in the plastic zone after tunnel excavation are solved, and the parameter sensitivity of the stress and displacement solutions is analyzed. The results show that the radius of plastic zone increases with the increase of excavation radius; when $m_i \leq 20$, r_p/r_0 increases nonlinearly with the increase of excavation radius; when $m_i = 25$, r_p/r_0 increases first and then decreases with the increase of excavation radius; at different m_i values, r_p/r_0 increases nonlinearly with the increase of GSI; among the effects of dilatancy and excavation radius on u/r_0 , the influence of excavation radius on displacement is more significant.

Key words: tunnel excavation; Hoek-Brown strength criterion; stress solution; displacement solution

隧道开挖破坏了隧址区的应力平衡, 导致围岩应力及位移场发生复杂变化, 引发围岩变形过大及支护体系开裂失稳等问题, 是地下工程研究的重点问题^[1-2]。

针对围岩应力及位移场复杂计算问题, 朱艳峰等^[3]提出将 Hoek-Brown 强度准则的应变软化模型应用于隧道塑性区分析中, 推导了隧道塑性区半径表达式。陆晓清等^[4]针对岩体开挖导致应力重分布问题, 以深埋轴对称圆形洞室为研究对象, 推导了理想弹塑性解析解。贺耕夫等^[5]依据 Hoek-Brown 破坏准则, 推导了寒区圆形隧洞理想弹塑性围岩的塑性区半径解析解。夏才初等^[6]基于三维非线性 Hoek-Brown 强度准则, 构建了考虑应变软

化特性的圆形隧道开挖后围岩非线性力学响应的求解方法。文献[7-8]基于 Hoek-Brown 本构模型, 推导了圆形隧道的弹塑性解, 探讨了其适用性。Massinas 等^[9]在双极坐标的基础上, 利用 Mohr-Coulomb 强度准则推导了圆形隧道围岩应力和塑性区的解析解。Exadaktylos 等^[10-11]推导了半圆形隧道的应力及位移计算表达式。李培楠等^[12]和施有志等^[13]分析了工程中常用的单心圆仰拱马蹄形隧道的解析解。程长清等^[14]根据水工隧洞运行期的受力情况, 运用 Mohr-Coulomb 与 Hoek-Brown 破坏准则, 对无限大均质体中圆形水工隧洞在有衬砌两种情况下的最小覆盖层厚度进行了弹塑性力学分析。

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综上所述, 针对服从 Mohr-Coulomb 准则时围岩的应力及位移解研究较多, 对 Hoek-Brown 准则下应力及位移的求解研究较为有限, 但在高围压条件下岩体节理裂隙发育, 此时 Hoek-Brown 强度准则更合适. 基于此, 本文利用非线性 Hoek-Brown 强度准则对深埋隧道开挖产生的弹塑性区进行求解, 探讨塑性区影响半径的影响因素, 并对特征参数进行敏感性分析, 以期对隧道开挖考虑非强度准则的应力及位移求解提供理论基础.

1 计算模型

构建无限大均质弹性体内圆形隧道开挖问题的简化计算模型, 如图 1 所示.

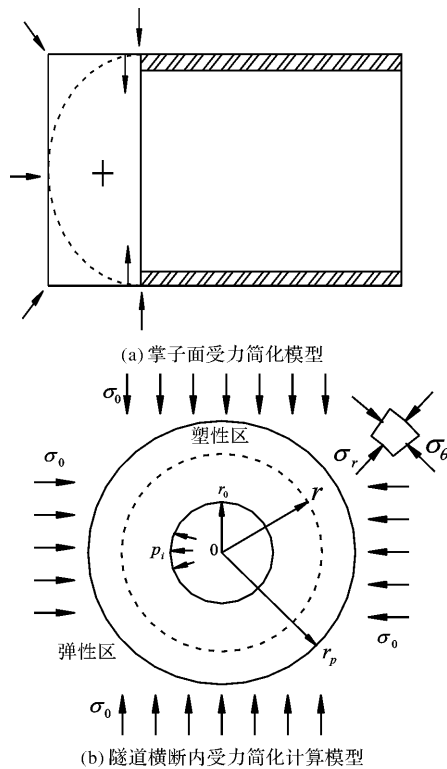


图 1 地下隧道开挖计算分析模型

Fig. 1 Calculation and analysis model of underground tunnel excavation

图中: σ_θ 、 σ_r 分别为环向应力及径向应力; σ_0 为作用在岩土体上初始静置土压力; p_i 为隧道开挖边界上支护压力; r_0 为隧道开挖的半径; r_p 为隧道开挖后围岩的塑性区半径.

2 Hoek-Brown 强度准则

Hoek-Brown 强度准则的屈服条件为

$$\sigma_\theta - \sigma_r = \sigma_c \left(m \frac{\sigma_r}{\sigma_c} + S \right)^a \quad (1)$$

式中: 系数 m 、 s 及 a 为表征岩石的基本特征的经验参数, 与参数 GSI 关系为

$$m = m_i \exp \left(\frac{GSI - 100}{28 - 14D} \right) \quad (2)$$

$$s = \exp \left(\frac{GSI - 100}{9 - 3D} \right) \quad (3)$$

$$a = \frac{1}{2} + \frac{1}{6} \left[\exp \left(-\frac{GSI}{15} \right) - \exp \left(-\frac{20}{3} \right) \right] \quad (4)$$

式(3)和(4)中 D 是岩石应力松弛程度或破坏程的参数, 取值在 0~1 之间; m_i 取值分别为 5, 10, 13, 16 和 24.

轴对称问题中, 岩体平衡微分方程为

$$\frac{d\sigma_r}{dr} + k \frac{\sigma_r - \sigma_\theta}{r} = 0 \quad (5)$$

式中, $k=1, 2$, 分别为洞身和掌子面附近区域, 即图 1 中的(a)与(b).

边界条件为: $\sigma_r|_{r=r_0} = p_i$, $\lim_{r \rightarrow \infty} \sigma_r = \sigma_0$.

3 应力及位移求解

3.1 弹性区中应力位移求解

考虑轴对称问题时, 弹性区中的应力和位移解为

$$\left. \begin{aligned} \sigma_r &= \sigma_0 + (\sigma_{rp} - \sigma_0) \left(\frac{R}{r} \right)^{k+1} \\ \sigma_\theta &= \sigma_0 - \frac{1}{k} (\sigma_R - \sigma_0) \left(\frac{R}{r} \right)^{k+1} \\ u &= \frac{1+\nu}{E} (\sigma_R - \sigma_0) \frac{R}{k} \left(\frac{R}{r} \right)^k \end{aligned} \right\} \quad (6)$$

式中, R 为影响区半径.

则弹塑性交界处的径向应变为

$$\epsilon_r^e = -k \epsilon_\theta^e = \frac{(1+\nu)}{E} (\sigma_R - \sigma_0) = B \quad (7)$$

将式(6)代入式(1), 求得弹塑性交界处的径向应力 σ_{rp} 的计算表达式为

$$(k+1)(\sigma_0 - \sigma_R) - k \sigma_c \left(m \frac{\sigma_R}{\sigma_c} + S \right)^a = 0 \quad (8)$$

式中, σ_0 为原岩应力.

式(8)可以通过 Newton-Raphson 法求解, 假定初始值 σ_{r0} , 则

$$\sigma_{Rn} = \sigma_{R0} + \frac{(k+1)(\sigma_0 - \sigma_{R0}) - k \sigma_c \left(m_b \frac{\sigma_{R0}}{\sigma_c} + S \right)^a}{(k+1) + k a m_b \left(m_b \frac{\sigma_{R0}}{\sigma_c} + S \right)^{a-1}} \quad (9)$$

3.2 塑性区中位移求解

在塑性区采用非关联流动准则

$$h \epsilon_r^p + k \epsilon_\theta^p = 0 \quad (10)$$

式中, 参数 h 与剪胀角 ψ 有关, 计算表达式为 $h = (1 + \sin\psi)/(1 - \sin\psi)$.

在塑性区, 环向和径向总应变包括弹性应变

和塑性应变两部分,表达式为

$$\epsilon_r = \epsilon_r^e + \epsilon_r^p \quad (11)$$

$$\epsilon_\theta = \epsilon_\theta^e + \epsilon_\theta^p \quad (12)$$

考虑小应变问题时,根据径向位移可求得应变表达式为

$$\left. \begin{aligned} \epsilon_\theta &= \frac{u}{r} \\ \epsilon_r &= \frac{du}{dr} \end{aligned} \right\} \quad (13)$$

据式(10~13)中的径向位移微分方程,可得

$$\frac{du}{dr} + \beta \frac{u}{r} = f(r) \quad (14)$$

式中, $f(r) = \epsilon_r^e + \epsilon_\theta^e$.

弹性塑性区交界处的径向位移边界条件为

$$u_c = \frac{c}{2G}(p_0 - p_{1y}) \quad (15)$$

式中, G 为围岩的剪切模量.

将式(15)代入(14),解得

$$u = \frac{1}{r^\beta} \int_c^r r^\beta f(r) dr + u_c \left(\frac{c}{r} \right)^\beta \quad (16)$$

为获得式(16)的积分值,可根据弹性应变的三种形式进行求解.

(1)第一种情况

假设在塑性区中弹性变形为定值,则弹性应变表达式为

$$\epsilon_r^e = -\frac{1}{2G}(p_0 - p_{1y}) \quad (17)$$

$$\epsilon_\theta^e = \frac{1}{2G}(p_0 - p_{1y}) \quad (18)$$

则函数 $f(r)$ 表达式为

$$f(r) = \frac{1}{2G}(\beta - 1)(p_0 - p_{1y}) \quad (19)$$

(2)第二种情况

考虑到塑性区内壁上 $r=a$ 受到内压力 $p_i - p_0$ 的作用,外部 $r=c$ 受到外部压力 $p_{1y} - p_0$ 的作用,可得弹性应变为

$$\epsilon_r^e = \frac{1}{2G} \left[(1-2v)C + \frac{D}{r^2} \right] \quad (20)$$

$$\epsilon_\theta^e = \frac{1}{2G} \left[(1-2v)C + \frac{D}{r^2} \right] \quad (21)$$

式中, v 为岩土体的泊松比;

$$C = \frac{(p_{1y} - p_0)c^2 - (p_i - p_0)a^2}{c^2 - a^2}; \quad D = \frac{a^2 c^2 (p_i - p_{1y})}{c^2 - a^2}.$$

此时, $f(r)$ 表达式为

$$f(r) = \frac{1}{2G} \left[(1-2v)(\beta + 1)C + (1-\beta)\frac{D}{r^2} \right] \quad (22)$$

(3)第三种情况

考虑静水压力时,弹性应变表达式为

$$\epsilon_r^e = \frac{1}{2G}[(1-v)(\sigma_r - p_0) - v(\sigma_\theta - p_0)] \quad (23)$$

$$\epsilon_\theta^e = \frac{1}{2G}[(1-v)(\sigma_\theta - p_0) - v(\sigma_r - p_0)] \quad (24)$$

此时函数 $f(r)$ 表达式为

$$f(r) = \frac{2}{2G}[(1-v-\beta v)\sigma_r + (\beta - v - \beta v)\sigma_\theta - (1-2v)(1+\beta)p_0] \quad (25)$$

根据 M-C 强度准则,求得函数 $f(r)$ 的表达式为

$$f(r) = \frac{1}{2G} \left[D_1 \left(\frac{r}{a} \right)^{a-1} + D_2 \right] \quad (26)$$

式中: $D_1 = (1+\beta)(1-2v)(A - p_0)$;

$D_2 = [(1-v-\beta v) + \alpha_r(\beta - \beta v - v)]B$.

求得式(16)中径向位移表达式为

$$\frac{u}{r} = \frac{1}{2Gr^{\beta+1}} [D_1 f_1(r) + D_2 f_2(r) + D_3 f_3(r) + 2Gu_c r^\beta - D_1 f_1(c) - D_2 f_2(c) - D_3 f_3(c)] \quad (27)$$

式中, $f_1(r) = \int r^\beta dr = \frac{r^{\beta+1}}{\beta+1}$;

$$f_2(r) = \int r^\beta \left(\frac{r}{a} \right)^{a-1} dr = \frac{r^{\beta+1}}{\beta+1} \left[\ln \left(\frac{r}{a} \right) - \frac{1}{\beta+1} \right].$$

3.3 塑性区应力场求解

将 Hoek-Brown 强度准则代入式(5)中,可得

$$\frac{d\sigma_r}{dr} - k \frac{\sigma_{ci}}{r} \left(m_b \frac{\sigma_r}{\sigma_{ci}} + S \right)^a = 0 \quad (28)$$

求解式(28),可得

$$\sigma_r = \frac{\sigma_{ci}}{m_b} \left\{ \left[\left(m_b \frac{p_i}{\sigma_{ci}} + S \right)^{1-a} + km_b(1-a) \ln \left(\frac{r}{r_0} \right) \right]^{\frac{1}{1-a}} - S \right\} \quad (29)$$

式中: r_0 为开挖轮廓线距离隧道中心的距离; p_i 为隧道支护反力.

将 Hoek-Brown 强度准则与式(29)联立,可获得环向应力的解析解.将 $\sigma_r = \sigma_R$ 代入式(29),可求得塑性区半径计算表达式为

$$R = r_0 \exp \left\{ \frac{1}{km_b(1-a)} \left[\left(m_b \frac{\sigma_R}{\sigma_{ci}} + S \right)^{1-a} - \left(m_b \frac{p_i}{\sigma_{ci}} + S \right)^{1-a} \right] \right\} \quad (30)$$

对于特殊情况 $k=1$ 时,式(29)~(30)则可简化为式(31)和式(32),即

$$\sigma_r = \frac{\sigma_{ci}}{m_b} \left\{ \left[\left(m_b \frac{p_i}{\sigma_{ci}} + S \right)^{1-a} + m_b(1-a) \ln \left(\frac{r}{r_0} \right) \right]^{\frac{1}{1-a}} - S \right\} \quad (31)$$

$$R = r_0 \exp \left\{ \frac{1}{m_b(1-a)} \left[\left(m \frac{\sigma_R}{\sigma_{ci}} + S \right)^{1-a} - \left(m_b \frac{p_i}{\sigma_{ci}} + S \right)^{1-a} \right] \right\} \quad (32)$$

4 算例分析

为验证本文理论的有效性,取文献

[15]的实验参数进行分析. 具体为: $r_0 = 5$ m, $D = 0$. 参数 a 、 m_b 和 S 的值则根据式(2)~(4)求得, 结果为: $a = 0.011$, $m_b = 5.128$, $S = 0.037$, 其他参数如表1所示.

表1 计算参数取值

Tab. 1 Calculation parameter values

参考 文献	岩石 质量	GSI	σ_{ci} /MPa	σ_{cir} /MPa	E /GPa	E_r /GPa
[15]	完好	75	150	150	42	10

求解参数 k 为不同参数的应力值, 绘制应力与塑性区半径的关系曲线, 如图2及图3所示.

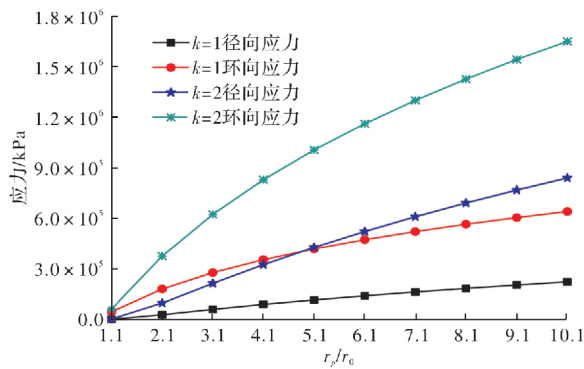
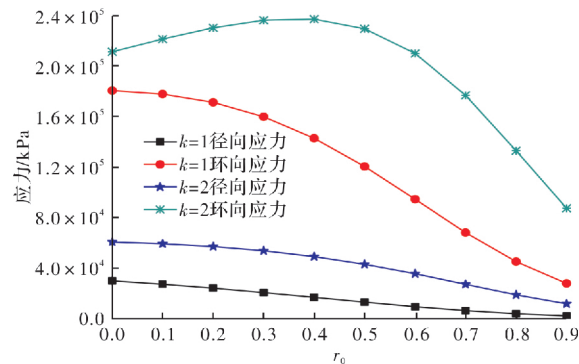
图2 应力与 r_p/r_0 的关系图Fig. 2 The relationship between stress and r_p/r_0 

图3 应力与开挖半径的关系

Fig. 3 The relationship between stress and excavation radius

图2及图3表明, 应力随着 r_p/r_0 的增加呈先增后减的趋势, 环向应力远小于径向应力, 开挖半径和 r_p/r_0 对环向应力的影响较径向应力显著.

开挖半径对塑性区半径的影响如图4所示, 不同 m_i 值时, r_p/r_0 与隧道开挖半径的关系如图5所示.

图4表明, 塑性区半径随着开挖半径的增大而增大. 图5显示, 当 $m_i \leq 20$ 时, r_p/r_0 随着开挖半径的增大非线性增大; 当 $m_i = 25$, r_p/r_0 随着开挖半径的增大呈现先增后减趋势, r_p/r_0 与 GSI 的关系如图6所示.

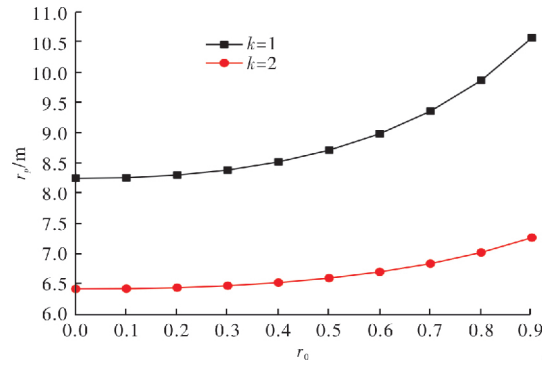
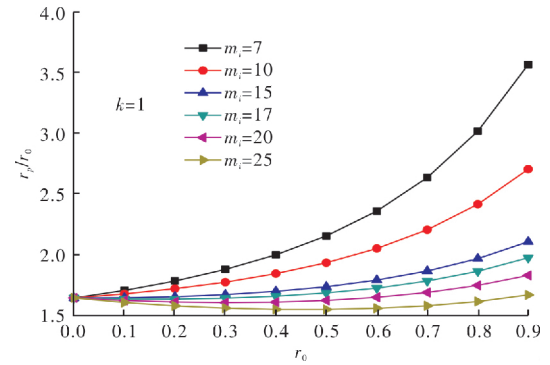
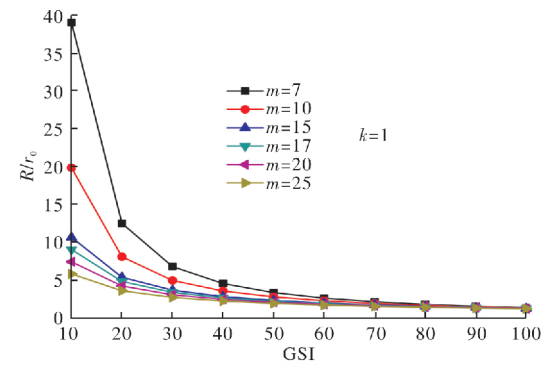
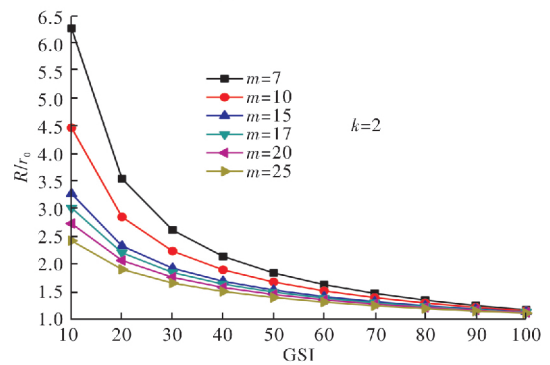


图4 塑性区半径与隧道开挖半径的关系

Fig. 4 The relationship between the radius of the plastic zone and the radius of tunnel excavation

图5 r_p/r_0 与隧道开挖半径的关系图Fig. 5 The relationship between r_p/r_0 and the radius of the tunnel excavation

(a) 掌子面附近区域



(b) 隧道横断面区域

图6 r_p/r_0 与 GSI 的关系图Fig. 6 The relationship between u/r_0 and GSI

图6表明, 不同 m_i 值时, r_p/r_0 随 GSI 增大

而非线性增大,其敏感性逐渐减弱。

利用本文推导公式,绘制 u/r_0 随着剪胀和开挖半径的分布规律如图7所示。

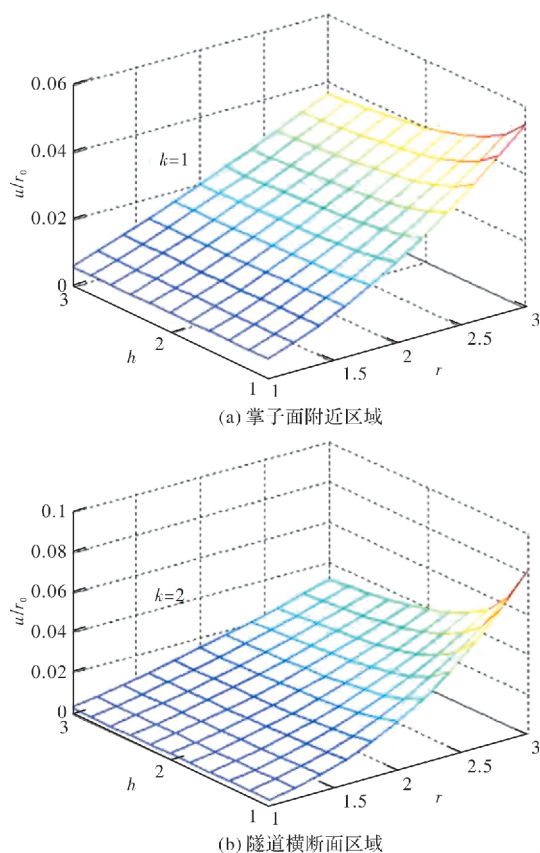


图7 u/r_0 与剪胀系数的关系图

Fig. 7 The relationship between u/r_0 and the dilatancy coefficient

由图7可知,剪胀和开挖半径对 u/r_0 的影响中,开挖半径对位移的影响更加显著。

5 结论

(1)本文基于非线性 Hoek-Brown 强度准则,求解了隧道开挖后塑性区的应力和位移解析解;

(2)利用本文推导公式,对影响应力和位移解的多个因素进行了特征参数敏感性分析,探讨了参数间的耦合影响及作用机制;

(3)塑性区半径随开挖半径的增大而增大;

(4)应力随塑性半径的增加呈现先增后减的演化趋势,同时环向应力远小于径向应力。

参考文献 References

- [1] 周光裕,谢小鱼,陈秋南,等. 基于最小安全系数法对隧道围岩稳定性分析[J]. 湖南科技大学学报(自然科学版), 2018, 33(3):54-58.
ZHOU Guangyu, XIE xiaoyu, CHEN Qiunan, et al. Stability analysis of the surrounding rockmass in tun-

- nel based on minimum safety factor method[J]. Journal of Hunan University of Science and Technology (Natural Science Edition), 2018,33(3):54-58.
- [2] LI Jun, SHAO Shengjun, SHAO Shuai. Collapsible characteristics of loess tunnel site and their effects on tunnel structure [J]. Tunnelling and Underground Space Technology, 2019, 83: 509-519.
- [3] 朱艳峰,张雪松,王和平. Hoek-Brown 强度准则的非圆形隧道塑性区半径预测[J]. 西安建筑科技大学学报(自然科学版), 2020,52(2):200-206.
ZHU Yanfeng, ZHANG Xuesong, WANG Heping. Application of Hoek-Brown strength criterion in stability analysis of non-circular tunnel[J]. J. of Xi'an Univ. of Arch. & Tech. (Natural Science Edition), 2020, 52(2): 200-206.
- [4] 陆晓清,魏良帅,黄海峰,等. 基于改进 Hoek-Brown 强度准则的深埋圆形洞室弹塑性分析[J]. 中国科技论文, 2020, 15(3):366-372.
LU Xiaoqing, WEI Liangshuai, HUANG Haifeng, et al. Elastoplastic analysis of deep buried circular chamber based on improved Hoek-Brown strength criterion [J]. China Science Paper, 2020, 15(3):366-372.
- [5] 贺耕夫,肖明砾,刘怀忠,等. 基于 Hoek-Brown 准则的寒区隧道围岩弹塑性分布影响因素分析[J]. 水电能源科学, 2020, 38(2): 130-134.
HE Gengfu, XIAO Mingli, LIU Huaizhong, et al. Analysis on influencing factors of elastoplastic distribution of surrounding rock in cold area tunnel based on hoek-brown criterion[J]. Water Resources and Power, 2020, 38(2): 130-134.
- [6] 夏才初,徐晨,刘宇鹏,等. 基于 GZZ 强度准则考虑应变软化特性的深埋隧道弹塑性解[J]. 岩石力学与工程学报, 2018, 37(11): 2468-2477.
XIA Caichu, XU Chen, LIU Yupeng, et al. Elastoplastic solution of deep buried tunnel considering strain-softening characteristics based on GZZ strength criterion[J]. Chinese Journal of Rock Mechanics and Engineering, 2018, 37(11): 2468-2477.
- [7] CAI M, KAISER P K, TASAKA Y, et al. Determination of residual strength parameters of jointed rock masses using the GSI system[J]. International Journal of Rock Mechanics and Mining Sciences, 2007,44(2): 247-265.
- [8] SQRENSEN ES, CLAUSEN J, DAMKILDE L. Finite element implementation of the Hoek-Brown material model with general strain softening behavior[J]. International Journal of Rock Mechanics and Mining Sciences, 2015, 78: 163-174.

(下转第 635 页)

- [10] CHEN D, YU X, GUO M, et al. Study on the mechanical properties of the mortars exposed to the sulfate attack of different concentrations under the triaxial compression with constant confining pressure [J]. *Construction and Building Materials*, 2017, 146(15): 445-454.
- [11] 韩宇栋, 张君, 高原. 混凝土抗硫酸盐侵蚀研究评述 [J]. *混凝土*, 2011(1): 52-56, 61.
HAN Yudong, ZHANG Jun, GAO Yun. Review of sulfate attack on concrete [J]. *Concrete*, 2011(1): 52-56, 61.
- [12] 苑立冬, 牛荻涛, 姜磊, 等. 硫酸盐侵蚀与冻融循环共同作用下混凝土损伤研究 [J]. *硅酸盐通报*, 2013, 32(6): 1171-1176.
YUAN Lidong, NIU Ditao, JIANG Lei, et al. Study on damage of concrete under the combined action of sulfate attack and freeze-thaw cycle [J]. *Bulletin of the Chinese Ceramic Society*, 2013, 32(6): 1171-1176.
- [13] 纪广. 超细微粉对水泥胶砂抗硫酸盐侵蚀的性能研究 [D]. 兰州: 兰州交通大学, 2020.
JI Guang. Study on the performance of super fine powder against sulphate erosion in cement sand [D]. Lanzhou: Lanzhou Jiaotong University, 2020.
- [14] JIN Zuquan, SUN Wei, JIANG Jinyang, et al. Damage of concrete attacked by sulfate and sustained loading [J]. *Journal of Southeast University*, 2008, 24(1): 69-73.
- [15] 梁咏宁, 袁迎曙. 硫酸盐腐蚀后混凝土单轴受压本构关系 [J]. *哈尔滨工业大学学报*, 2008, 40(4): 532-535.
LIANG Yongning, YAN Yingshu. Constitutive relation of sulfate attacked concrete under uniaxial compression [J]. *Journal of Harbin Institute of Technology*, 2008, 40(4): 532-535.
- [16] XIAO Dong, JIANG Guanlu, LIAO Dan, et al. Influence of cement-fly ash-gravel pile-supported approach embankment on abutment piles in soft ground [J]. *Journal of Rock Mechanics and Geotechnical Engineering*, 2018, 10(5): 977-985.

(编辑 沈波)

(上接第 629 页)

- [9] MASSINAS S A, SAKELLARIOU M G. Closed-form solution for plastic zone formation around a circular tunnel in half-space obeying Mohr-Coulomb criterion [J]. *Geotechnique*, 2010, 60(7): 569-571.
- [10] EXADAKTYLOS G E, STAVROPOULOU M C. A closed-form elastic solution for stresses and displacements around tunnels [J]. *International Journal of Rock Mechanics & Mining Sciences*, 2002, 39: 905-916.
- [11] EXADAKTYLOS G E, LIOLIO P A, STAVROPOULOU M C. A semi-analytical elastic stress displacement solution for notched circular openings in rocks [J]. *International Journal of Solids and Structures*, 2003, 40: 1165-1187.
- [12] 李培楠, 刘俊, 苏锋, 等. 任意形状隧道围岩应力与位移的解析延拓求解 [J]. *同济大学学报(自然科学版)*, 2013, 41(10): 1483-1489.
LI Peinan, LIU Jun, SU Feng, et al. Analytical continuation method for solving stress and displacement of surrounding rock buried tunnel excavation with arbitrary shape section [J]. *Journal of Tongji University (Natural Science)* 2013, 41(10): 1483-1489.
- [13] 施有志, 高轩能. 深埋马蹄形隧道开挖围岩应力与位移的复变函数解 [J]. *土木建筑与环境工程*, 2014, 36(1): 101-105, 118.
SHI Youzhi, GAO Xuanneng. Complex variable function solution of stress and displacement of surrounding rock buried deep horseshoe-shaped tunnel excavation [J]. *Journal of Civil Architectural and Environmental Engineering*, 2014, 36(1): 101-105, 118.
- [14] 程长清, 杨自友, 殷海波, 等. 有压隧洞围岩最小覆盖层厚度弹塑性力学分析 [J]. *地下空间与工程学报*, 2021, 17(2): 413-420.
CHENG Changqing, YANG Ziyu, YIN Haibo, et al. Elastoplastic analysis on the minimum covering layer thickness of the pressure-tunnel surrounding rock [J]. *Chinese Journal of Underground Space and Engineering*, 2021, 17(2): 413-420.
- [15] HOEK E, BROWN E T. Practical estimates of rock mass strength [J]. *International Journal of Rock Mechanics & Mining Sciences*, 1997, 34(8): 1165-1186.

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